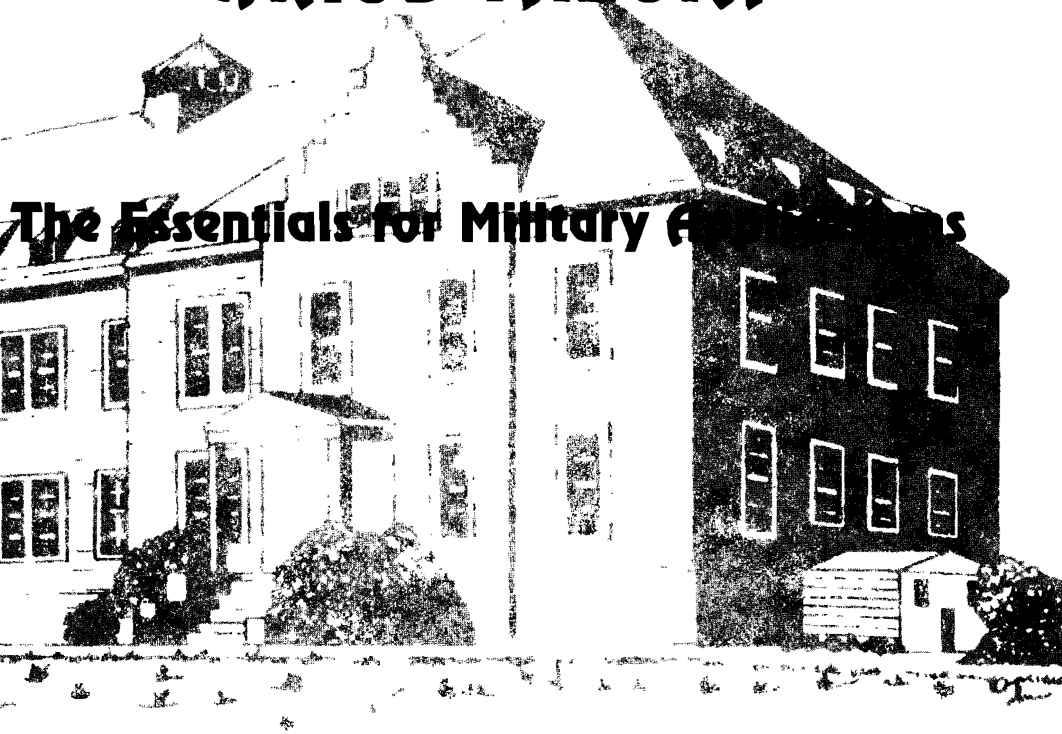
The Newport Papers

*Tenth in the Series*

# CHAOS THEORY

## The Essentials for Military Applications



Glenn E. James
Major, U.S. Air Force

NAVAL WAR COLLEGE

# Chaos Theory

Naval War College
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# Chaos Theory

## The Essentials for Military Applications

Glenn E. James
Major, U.S. Air Force

§

Printed in the United States of America

to my patient family

Patricia Ann
Christine Marie
Phillip Andrew

who continue to follow me
bravely into Chaos

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# Foreword

I take great pleasure in presenting a remarkable addition to our Newport Papers series. While Major Glenn E. James, the author, received support and assistance from sources within his own service, the U.S. Air Force, the final research and the paper itself are the products of his term in the Advanced Research Program at the Naval War College. This paper typifies the quality of work and capabilities of our students from all the services here at the College. It is an excellent example of the benefits we derive from the close collaboration between our academic and research departments.

*Chaos Theory: The Essentials for Military Applications* is a highly challenging work, one which demands—but amply repays—close attention. It asks for imagination to envision clearly the military applications for which the author argues. Major James hopes that his efforts can help those of us who labor in the field of national security to appreciate that Chaos theory is a valuable discipline. While many of the applications of this new field remain conjectural and as yet unclear, Major James has written a pioneering work which invites military officers to understand the principles of Chaos and to look for applications. I commend this Newport Paper in particular to policy-level readers, who will find it a useful and understandable overview of the subject, and to the faculty members of all of the service war colleges for whom we offer this as a useful text.

J.R. Stark
Rear Admiral, U.S. Navy
President, Naval War College

# *Preface*

## Before You Begin...

Before you start into this report, it may help to relax and to prepare to be patient.

## Be Patient with the Material...

Chaos as a branch of mathematics is still very young. The first concrete results surfaced only thirty years ago. Enormous opportunities for new research remain unexplored. As of yet, not all the bodies of interested researchers know one another or exchange (or search for) information across disciplinary lines. This paper represents my effort to continue the published conversation on Chaos applications. I'm inviting you to eavesdrop, because the issues are crucial to the military profession.

## Be Patient with the Essay...

Several officers learned of my background in mathematics, and as I left for the Naval War College, they asked me to consider how Chaos theory influences the military profession. I examined the published resources that were being used and felt compelled to correct some serious errors. Many publications overlook key results, make fundamental technical mistakes, or scare the reader with the complexity of the issues. While the progress documented in those papers is noteworthy—many well-intentioned efforts were made under severe time constraints—we are overdue for a mid-course correction to prevent the errors from propagating further.

My own Chaos research began in 1987 in my Ph.D. studies at Georgia Tech, where Professor Raj Roy introduced me to Chaos in lasers. Since then, I have taught mathematics for four years at the Air Force Academy, including three special topics courses on Fractals and Chaos. This past year, I gave formal presentations to the Air Command and Staff College student body and to two small seminars of Naval War College faculty. This paper grew out of those talks, subsequent questions, and my continuing research.

I have aimed this report at the broad population of students attending the various war colleges. I have made the format conversational so I may talk *with* them, not *at* them, since this essay takes the place of what I might discuss in a

more personal, seminar environment. I struggled to strike a useful balance, sometimes offering many examples so that I can reach this broad audience, and at other times foregoing extended illustrations on behalf of brevity. I have assumed a *minimal* technical background, and resort to an appendix only where absolutely necessary. I also offer a substantial bibliography of what I consider to be the best available references for the reader who is anxious for more.

## Be Patient with Yourself...

Finally, relax. Chaos isn't hard to learn—it's only hard to learn *quickly*. The important results are often abstract generalizations, but we can arrive at those conclusions via examples and demonstrations that are not difficult to *visualize*. Allow yourself to *wonder*.

In his splendid book, *Fractals Everywhere*, Michael Barnsley warns:

> There is a danger in reading further. You risk the loss of your childhood vision of clouds, forests, galaxies, leaves, feathers, . . . and much else besides. Never again will your interpretation of these things be quite the same.[1]

I will also warn you of the risks of *not* reading further: you may fail to understand phenomena that are *essential* to decision makers, particularly in an era when the speed and volume of feedback can drive the dynamics of our physical and social—hence, our military—systems into Chaos.

# *Acknowledgements*

My thanks go first to Colonel John Warden, U.S. Air Force, Commandant of the Air Command and Staff College (ACSC), who first asked me to consider this research topic. I gratefully acknowledge the Center for Naval Warfare Studies at the Naval War College, which sponsored my work under the Advanced Research Program, and Lieutenant Colonel Roy Griggs in the Air Force Strategic Planning office, who sponsored this work along with Lieutenant Colonel Jeff Larson through the USAF Institute for National Security Studies at the U.S. Air Force Academy.

I am greatly indebted to my friend Major Bruce DeBlois, U.S. Air Force, Professor of Air and Space Technology at the School for Advanced Airpower Studies, Maxwell Air Force Base, Alabama, who fought hard to incorporate this research into the Air University curriculum, and to Lieutenant Colonel Rita Springer and Lieutenant Colonel Gus Liby, U.S. Air Force, for arranging my first of several presentations to the ACSC student body.

I especially appreciate the extensive research and the encouragement of my friend, Commander Bill Millward, U.S. Navy. He contributed a vast amount of preliminary research and references that made this project possible in the Advanced Research trimester at the Naval War College.

My thanks go also to many individuals who patiently critiqued my seminars and drafts: the Naval War College faculty in Strategy and Force Planning; the Rational Decision Making faculty; Dr. John Hanley, Program Director of the Strategic Studies Group, and the students in his Decision Theory Elective; Lieutenant Colonel Lloyd Rowe, U.S. Air Force; and Major Tom Tomaras, U.S. Air Force. Finally, my deepest thanks to Dr. Stephen Fought, my advisor, for his brave support and constant encouragement.

# *Executive Summary*

This paper distills those features of Chaos theory essential to military decision makers. The new science of Chaos examines behavior that is characterized by erratic fluctuations, sensitivity to disturbances, and long-term unpredictability. This paper presents specific ways we can recognize and cope with this kind of behavior in a wide range of military affairs.

Designed for courses at the various war colleges, the paper makes three new contributions to the study of Chaos. First, it reviews the fundamentals of chaotic dynamics; the reader needs no extensive prerequisites in mathematics. Much more than a definition-based tutorial, the first part of this paper builds the reader's intuition for Chaos and presents the essential consequences of the theoretical results. Second, the paper surveys current military technologies that are prone to chaotic dynamics. Third, the universal properties of chaotic systems point to practical suggestions for applying Chaos results to strategic thinking and decision making. The power of Chaos comes from this universality: not just the vast number of chaotic systems, but the common types of behaviors and transitions that appear in completely unrelated systems. In particular, the results of Chaos theory provide new information, new courses of action, and new expectations in the behavior of countless military systems. The practical applications of Chaos in military technology and strategic thought are so extensive that every military decision maker needs to be familiar with Chaos theory's key results and insights.

# Introduction

## *Welcome and Wonder*

---

> Physicists, mathematicians, biologists, and astronomers have created an alternative set of ideas.
> Simple systems give rise to complex behavior.
> Complex systems give rise to simple behavior.
> And most important, the laws of complexity hold universally, caring not at all for the details of a system's constituent atoms.[2]
>
> James Gleick

---

### Wake Up and Smell the Chaos

The contractor for the operational tests of your new missile system has just handed you the chart in figure 1. He ran two tests, identical to six decimal places, but the system performance changed dramatically after a few time-steps. He thinks there was a glitch in the missile's telemetry or that somebody made a scaling error when they synthesized the data. Could it be that the data is correct and your contractor is overlooking something critical to your system?

Your wargaming staff is trying to understand and model the time dependence of American aircraft losses in Vietnam. They look at the data in figure 2 and quit. It's just a random scatter of information, right? No patterns, no structure, too many variables, too many interactions between participants, too large a role played by chance and human choice. No hope, right?

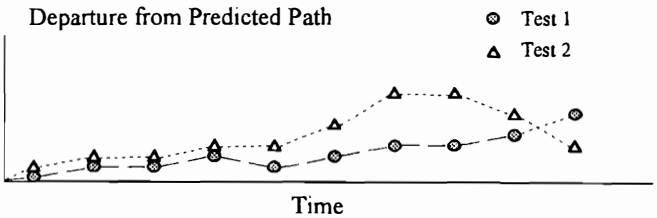



*Figure 1. Hypothetical Missile Test Data: What Went Wrong?*

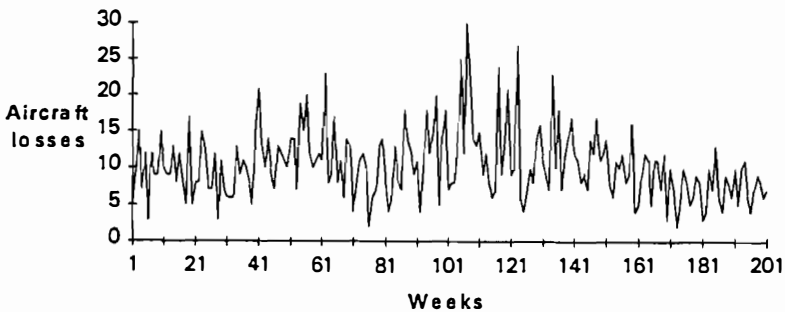



*Figure 2. U.S. Aircraft Losses in Vietnam.*[3]

The results of the new science of Chaos theory offer some intriguing answers to questions like these. Moreover, the theory has profound implications for the dynamics of an enormous variety of military affairs. In fact, the applications of Chaos in military technology and strategic thought are so extensive that every military decision maker needs to be familiar with Chaos theory's key results and insights.

## Why Chaos with a Capital "C"?

Chaos, as discussed here, is *not* social disorder, anarchy, or general confusion. Before you read on, set aside your connotations of the social (small "c") chaos

reported on the evening news. Chaos is a relatively new discipline of mathematics with boundless applications; to highlight the difference, I will keep this special use capitalized throughout.

Chaos theory describes a specific range of irregular behaviors in systems that move or change. What is a system? To define a system, we need only two things: a collection of elements—components, players, or variables—along with a set of rules for how those elements change—formulas, equations, recipes, or instructions.

A remarkable feature of chaotic change is its contrast with "random" motion. We generally label as random many irregular changes whose dynamics we can not predict. We will find, as this report progresses, that Chaos displays many of the same irregularities, with one important difference: the apparently random motion of a chaotic system is often described by *completely deterministic equations of motion*! Several specific examples of chaotic systems in this will illustrate this point.

The term "Chaos" was first applied to such phenomena fewer than thirty years ago—that's a *hot* topic for mathematics! James Yorke characterized as "chaotic" the apparently unpredictable behavior displayed by fluid flow in rivers, oceans, and clouds.[4] Today, artificial systems move and react fast enough to generate similar, erratic behavior, dynamics that were seldom possible before the advent of recent technologies. Nowadays, many military systems exhibit Chaos, so we need to know how to recognize and manage these dynamics. Moreover, the universality of many features of Chaos gives us opportunities to exploit these unique behaviors. Learn what to expect. This is not a fleeting fad: **real systems** really behave this way.

## What's New in This Essay?

Although numerous Chaos tutorials are available in various disciplines, there are three main deficiencies in the available resources:

- Many tutorials require an extensive background in mathematical analysis.
- Many works do not focus on useful *applications* of Chaos theory; they simply offer a smorgasbord of vocabulary and concepts.
- Some contain major technical flaws that dilute their potential application or mislead the reader.

So, the immediate need is threefold: we require an accessible bridge to connect us with the basis of Chaos theory; we seek some in-depth demonstrations of its applications; and we must avoid fundamental conceptual errors.

## Who Cares?

Even if Chaos can help military *analysts*, why should *everyone* be exposed to the theory? After all, there is a balance here—you don't need to know quantum physics to operate a laser printer, right? This paper will show that Chaos occurs in virtually every aspect of military affairs. The 1991 Department of Defense (DOD) Technologies Plan, for instance, set priorities for research spending.[5] It ranked the following technologies based on their potential to reinforce the superiority of U.S. military weapon systems:

1. semiconductor materials and microelectronics circuits
2. software engineering
3. high-performance computing
4. machine intelligence and robotics
5. simulation and modeling
6. photonics
7. sensitive radar
8. passive sensors
9. signal and image processing
10. signature control
11. weapon system environment
12. data fusion
13. computational fluid dynamics
14. air-breathing propulsion
15. pulsed power
16. hypervelocity projectiles and propulsion
17. high-energy density materials
18. composite materials
19. superconductivity
20. biotechnology
21. flexible manufacturing

Every one of these technologies overlaps with fundamental results from Chaos theory! In particular, the chaotic dynamics possible in many of these systems arise due to the presence of *feedback* in the system; other sources of Chaos are discussed elsewhere in this report. In this paper, you will discover that the fundamentals of Chaos are as important to military systems as Newton's laws of motion are to classical mechanics.

Numerous systems tend to behave chaotically, and the military officer who does not understand Chaos will not understand many of the events and processes

that mark the life of today's competent military professional. Look again at figure 2. Not too long ago, if we had measured output like figure 2 in any scenario, our analysts would have packed up and gone home, dismissing the data as random noise. However, it is not "noise" at all. Chaos theory helps us to know when erratic output like that in the figure may actually be generated by deterministic (non-random) processes. In addition, the theory provides an astounding array of tools which make *short-term predictions* of the next few terms in a sequence, predict *long-term trends* in data, *estimate how many variables* drive the dynamics of a system, and *control dynamics* that are otherwise erratic and unpredictable. Moreover, this analysis is often possible without any prior knowledge of an underlying model or set of equations.

Applied Chaos theory already has a growing community of its own, but the majority of military decision makers are not yet part of this group. For example, the Office of Naval Research (ONR) leads DOD research into Chaos applications in engineering design, but more military leaders need to be involved and aware of this progress. Beyond the countless technical applications, many of which readily translate to commercial activities, we must concern ourselves with strategic questions and technical applications that are unique to the profession of warfare. Chaos theory brings to the table practical tools that address many of these issues.

## *Why Now?*

As long as there has been weather, there have been chaotic dynamics—we are only now beginning to understand their presence. Some scientists, like Jules-Henri Poincaré in the late 1800s, had inklings of the existence of Chaos, but the theory and the necessary computational tools have only recently matured sufficiently to study chaotic dynamics. In 1963 Edward Lorenz made his first observations of Chaos quite by accident when he attempted simulations that had become possible with the advent of "large" computers. Currently, high-speed communications, electronics, and transportation bring new conduits for feedback, driving more systems into Chaos. Consider, for instance, the weeks required to cross the Atlantic to bring news of the American Revolution to Britain. Now, CNN brings updates to decision makers almost instantaneously.

Until recently, observations of the irregular dynamics that often arise in rapidly fluctuating systems have been thrown away. Unless we train decision makers to look for specific dynamics and the symptoms of imminent behavior transitions, erratic data sets will continue to be discarded or explained away.[6]

## Clear Objectives

As the preface suggested, Chaos theory is not difficult to learn—it is only difficult to learn *quickly*. Am I violating this premise by trying to condense the essentials of Chaos into this single paper? I hope not. I am trying to build a bridge and sketch a map. The bridge spans the gap that separates physical scientists on one side—including analysts in mathematics, physics, and electrical engineering—and humanists on the other—experts in psychology, history, sociology, and military science. The bibliographical map identifies specific references on issues that interest segments of the broad audience that I hope to reach with this paper.

My intent here is to teach the reader just enough to be dangerous, to highlight the places where Chaos happens all around us. The results of Chaos theory can improve military decision making and add new perspectives to creative thought. I also will offer enough examples and applications so that readers can recognize chaotic dynamics in common situations. Eventually, I hope the reader will grasp the key results and apply them in various disciplines. My ultimate aim is to offer a new perspective on motion and change, to heighten your curiosity about Chaos, and to provide adequate tools and references to continue the deeper study that is essential to fully understanding the fundamentals of Chaos.

Here's the plan. In chapter I we start with Chaos that can be demonstrated at home, so skeptics will believe Chaos is more than a metaphor, and so we all can visualize and discuss important issues from a common set of experiences. I do not want you mistakenly to believe that you need access to high-technology circuits and lasers to concern yourself with Chaos—quite the contrary. Then we'll add some detail in chapter II, complementing these intuitions with better definitions. In chapter III, we consider the pervasiveness of Chaos in military systems. Chapter IV offers practical means for applying Chaos theory to military operations and strategic thinking. Most of the discussions proceed from specific to general in order to lend a broad perspective of how Chaos gives *new information, new options for action, and new expectations of the dynamics* possible in military systems.

In the end, I hope you will learn to:

- *Recognize chaos* when you encounter it;
- *Expect chaos* in your field, your organization, and your experiments; and,
- *Exploit chaos* in your decision making and creative thought.

# *Part One*

## What IS Chaos?

---

Somehow the wondrous promise of the earth
is that there are things beautiful in it,
things wondrous and alluring,
and by virtue of your trade
you want to understand them.[7]

Mitchell J. Feigenbaum

---

# I

# Demonstrations

The disorderly behavior of simple systems . . .
generated complexity:
richly organized patterns,
sometimes stable and sometimes unstable,
sometimes finite and sometimes infinite,
but always with the fascination of living
things.
That was why scientists played with toys.[8]

James Gleick

## DEFINITELY Try This at Home!!!

The simple demonstrations in this chapter offer visualizations of a wide range of chaotic dynamics. They also provide a good introduction to the methods and tools available to observe, measure, and analyze these dynamics. My main goal is to build the reader's *intuition* of what Chaos looks like.

For any skeptical reader, these examples represent the first exhibits of the extensive evidence I will produce to demonstrate how prevalent chaotic dynamics are. For all readers, this chapter outlines common examples that provide a context useful for discussing definitions, tools, key results, and applications in subsequent chapters. We begin with demonstrations to set up at home in order to show that access to high technology is not needed to observe Chaos. Quite the

contrary: Chaos arises in some of the simplest physical systems. This brief exposure to chaotic dynamics may also spark imagination about the systems where Chaos may operate in particular areas of interest. A little later, after a more complete description of the vocabulary and tools of Chaos (chapter II), we will examine the *military* systems where one should expect to see Chaos (chapter III).

Remember: as we work through each example, the reader should gradually come to expect and recognize Chaos in *any* system that changes or moves. As a general plan for each demonstration that follows, we will:

1. Describe the physical system and answer clearly:
   What is the system?
   What is being measured?
2. Preview the significance of the particular system:
   Why do we care about *this* demonstration?
   Does it relate to any military system?
3. Discuss the significant dynamics and transitions.
4. Highlight those results and characteristics common to many chaotic systems.

The answers to item 1 are crucial. The confusion in many discussions about Chaos can be traced to a failure to identify either a well-defined system or a set of measurements. Likewise, to understand the appropriate ways to *apply* the insights of Chaos, we need to use its terminology with some care. With this priority in mind, the discussion of each demonstration will offer a first peek at the Chaos vocabulary that chapter II presents in greater detail.

## Warm-ups with a Simple Pendulum

Before we exercise our imaginations with chaotic dynamics that may be entirely new, let's first "stretch out" by examining the detailed behavior of a pendulum. The simplicity of this example makes it easy to visualize and to reconstruct in your home or office; it gives us an indication of good questions to ask when we observe other systems.

As a hint of things to come, an extraordinary number of complicated physical systems behave just like a pendulum, or like several pendulums that are linked together. Picture, for instance, a mooring buoy whose base is fixed to the sea floor but whose float on the surface (at the end of a long slack chain) is unconstrained. Much of the buoy's motion can be modeled as an upside-down pendulum.[9]

What is the pendulum's "system," exactly? A fixed mass, suspended at the end of a rigid bar, swings in only two dimensions (left and right swings only, no

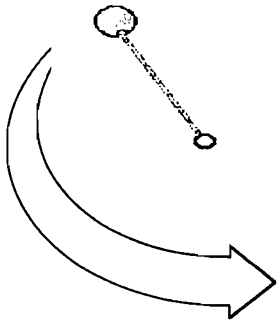


*Figure 3. Simple Pendulum, No Wobbles Allowed!*

additional motion). The end of the bar is fixed at a single point in space, but let us assume there is no "ceiling," so the pendulum is allowed to swing up over its apex and around to the other side (figure 3). Notice that as we define the system we must clearly state our *assumptions* about its components and its behavior.

What can be observed and measured in this system? Fortunately, in this example we need only two pieces of information to describe completely the physical "state" of the system: *position* and *velocity*. The pendulum's position is measured in degrees; its velocity is measured in degrees per second. These two observable quantities are the only two independent variables in the system, sometimes referred to as its *degrees of freedom* or *phase variables*. A system's phase variables are those time-dependent quantities that determine its state at a given time. Observe that even though the pendulum swings in a curve that sits flat in a two-dimensional plane, we need only one variable to describe the pendulum's position in space. Therefore, the pendulum has only *one degree of freedom* in its angular position.

So, what can this pendulum do? Let's pretend, at first, that it experiences no friction, drag, or resistance of any kind. This ideal pendulum exhibits a rich variety of behavior. If we start it at "the bottom," where both position and velocity are zero, it stays there. Any state that has this property—not changing or moving when undisturbed—is called an *equilibrium, steady state*, or *fixed point* for the system. If we displace the pendulum a few degrees to either side and just let it go, it swings back and forth *periodically*, with the same amplitude, forever. In this ideal system, we can also carefully balance the pendulum at the top of its swing,

and it will stay put forever. This state, with position 180 degrees and velocity zero, is another *equilibrium point*.

Does this ideal pendulum display any other dynamics? Perhaps just one more: we can impart enough *initial velocity* to the pendulum so that it swings upward over its apex and continues to wrap around its axle, forever. This completes the list of possible dynamics for the ideal pendulum, and it completes a first exposure to some important terms used to describe all *dynamical systems*.

Now let's get back to reality and add some resistance to the system, where the pendulum experiences "damping" due to friction and drag. This real pendulum still has the same two equilibrium points: the precise top and bottom of its swing, with zero velocity. A new feature we can discuss, though, is the *stability* of these *fixed points*. If we disturb any pendulum as it hangs at rest, it eventually slows its swing and returns to this low equilibrium. Any such fixed point, where small disturbances "die out," and the system always returns to its original state, is called a *stable fixed point* (figure 4a). On the other hand, at the top position of 180 degrees, any perturbation to the right or left sends the pendulum into a brisk downswing that eventually diminishes

(a) Ahhh . . . stability (b) Unstable . . Don't Breathe !!

*Figure 4. Stable and Unstable Equilibrium Points.*

until the pendulum hangs at rest. When a system tends to depart from a fixed point with any minuscule disturbance, we call it an *unstable fixed point* (figure 4b).

We should note several other issues concerning the pendulum's motion that will arise when we study more complicated systems. First is the observation that the pendulum (with friction) displays both *transient* and *limit dynamics*. The *transient dynamics* are all the swings, which eventually damp out due to resistance in the environment. After all the transients die out, the system reaches its *limit*

*dynamics*, which in this case is a single state: the lower fixed point, with zero position and velocity.

It seems we may be reaching the point where we have exhausted the possible dynamics in this simple pendulum system. After all, even though this is a harmless way to introduce the vocabulary of fixed points, dynamics, transience, and stability, there is only so much a pendulum can do. Right?

When the system remains undisturbed, the answer is a resounding Yes! The reason: the motion of a simple pendulum, unforced, is a *linear system* whose solutions are all known. In particular, the equation of motion, for the position $y$, comes from Newton's familiar relation, force equals mass times acceleration:

$$my'' + cy' + ky = 0, \tag{1}$$

where $m$ is the pendulum mass, $c$ is a measure of friction in the system, and $k$ includes measures of gravity and bar length.

Now, the swinging motion we *observe* appears to be anything but linear: a pendulum swings in a curve through space, not a straight line, and the functions that describe oscillations like these are wavy sines and cosines. However, the *equation of motion*—like the system itself—is called *linear* because the equation consists of only *linear operations*: addition, multiplication by constants, and differentiation. When the pendulum experiences no external forces, the resulting *homogeneous* equation shows a zero on the right-hand side of equation (1). What is the significance of recognizing a linear, homogeneous system? *All* the solutions are *known*; all possible behaviors are known and predictable.

To add the last essential layer of reality and to generate some interesting motion in the pendulum system, envision a playground swing. Once you start yourself swinging, how do you get yourself to swing much higher? You add a relatively small external force to the system: you kick your legs and lean forward and back in a manner carefully timed with the larger motion of the swing itself. By pumping your legs like this, you add a periodic force to the right side of equation (1) and you resonate and amplify a natural frequency of the large swing.

This addition of an external kick, or forcing function, to the pendulum system can induce interesting new dynamics. Be aware that, especially if you are pushing someone else on the swing, you can control three different features of the perturbation: where you push, how hard, and how often. The system may respond to the external forcing in many different ways. It may resonate with one of its natural frequencies (you may have seen the film of the Tacoma Narrows Bridge being destroyed by the violent oscillations induced by resonance with wind

gusts). In another instance, the swing may behave quite unpredictably if you push the chains instead of the swing. You may momentarily bring the entire system to a halt, or cause intermittent lurches in the swing; or you may get very regular motion for a long time, only occasionally interrupted by off-cycle bumps or jostles.

The playground swing, as a system, is just like the simple pendulum. However, when you "kick" it occasionally, you begin to observe departures from predictable behavior. This irregular sort of behavior, characteristic of a kicked pendulum, is one of the many traits of Chaos: behavior that is *not periodic*, *apparently random*, where the *system response is still recurrent* (the pendulum still swings back and forth) but *no longer in a predictable way*. In his classic work on Chaos, James Gleick correctly asserts that, because of the rich dynamics possible in such a simple system, physicists were unable to understand turbulence or complexity accurately until they understood pendulums. Chaos theory unites the study of different systems so that the dynamics of swings and springs broaden to bring new insights to high technologies, from lasers to superconducting Josephson junctions, control surfaces in aircraft and ships, chemical reactions, the beating heart, and brain wave activity.[10]

As this paper continues, we will see more detailed connections between the behavior of pendulums and other more complicated systems. For now, let us move on to our second home demonstration of Chaos, introduce some additional vocabulary, and continue to build our intuition for what we should expect to see in a chaotic system.

## The Dripping Faucet

The second home demonstration can be done at the kitchen sink, or with any spout where you can control the fluid flow and observe individual drops. This demonstration mimics an original experiment by Robert Shaw and Peter Scott at the University of California Santa Cruz.[11] It wonderfully illustrates several features of Chaos, particularly the *transitions* between various dynamics, which are common to many systems. The results are so astounding that you may want to bring your reading to the sink right now and experiment as you read along. Otherwise, you may not believe what you read.

What is the system? To recreate the Santa Cruz experiment, we need a faucet handle or spigot that can be set at a slow flow rate and then be left alone so we can observe drops emerging for a few minutes. We need enough water available so the flow continues without interruption. Finally, we need some means to detect

the time intervals between drops. We don't need a stopwatch, exactly, but we do need a clear view of the drops, or we need the drops to land on some surface that resounds loudly enough for us to detect patterns and rhythms as the drops fall. Fortunately, we need no assumptions about the water quality or any details about the size or material of the spout. We just need drops.

What can we observe and measure in this system? We want to have a clear view of the drops forming; this will give us some intuition as to *why* the flow makes transitions between different kinds of behavior. We want to measure the *time intervals* between drops. Shaw and Scott did this very precisely with a laser beam and computer. For us, it will suffice to watch or listen as the drops land.

What's the significance? Because of the difficulties in modeling any fluid, there is absolutely no hope of simulating even a single drop forming and dropping from a faucet. However, by measuring only one simple feature of the flow, the time between events, we can still understand many characteristics of the system dynamics. We will observe, for example, specific transitions between behaviors, transitions that are common to many chaotic systems. We will also gain some useful metaphors that are consistent with our intuitions of human behavior; but, much more important, we will learn some specific *things to expect* in chaotic systems, even when we cannot model their dynamics.

So, what kinds of things can a sequence of water drops do? If the spigot is barely open and the flow extremely slow, you should observe a slow, regular pattern of drips. Leave the faucet alone, and the steady, aggravating, *periodic* rhythm will continue far into the night. This pattern represents *steady state, periodic* output for this system. Increase the flow ever so slightly, and the drips are still periodic, but the *time interval* between drips decreases, that is, the frequency increases. At the other extreme of its behavior, with the flow rate turned much higher, the water will come out as a steady, unbroken stream. No real excitement yet.

The big question is: What happens *in between* these two extreme behaviors? How does the flow make its transition from periodic drips to a steady stream? We'll move step by step through the transitions in this system. Low flow rates will generate slow, regular drips. Increased flow will produce regular drips with new patterns. After a certain point, the drop dynamics will prevent the faucet from dripping regularly, and we will see evidence of Chaos.

Here's how to proceed with the experiment. Start with slow, steady dripping. Watch, for a moment, how the drops form. A single drop sticks to the end of the spout and begins to fill with water, like the elastic skin of a balloon (figure 5).

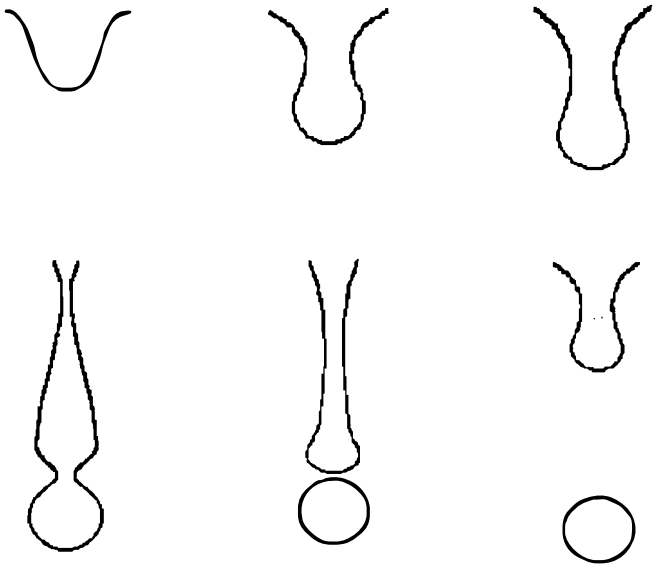


*Figure 5. Formation of Water Drops from a Spout.*

Eventually the drop grows large enough to overcome its surface tension; it breaks off and falls. The water left on the spout first springs back and recovers, then it begins to fill up to form the next drop: we will see that the time it takes to do all this is a critical feature of the system.

Now, *very* gradually, increase the flow rate. For a while, you will still see (or hear) periodic dripping, while the frequency continues to increase. However, before too long—and before the flow forms a solid stream—you will observe a different dripping pattern: an irregular pattern of rapid dripping interspersed with larger splats of various sizes, all falling at erratic, unpredictable time intervals. What causes the new behavior? The drops are beginning to form so quickly that a waiting drop does not have time to spring back and completely recover before it fills with water and breaks off. This is *chaotic* flow.

This deceptively simple demonstration is essential to our intuition of Chaos, for several reasons. First, despite the nasty fluid physics that is impossible to model in detail, we are able to make simple measurements of time intervals and learn a great

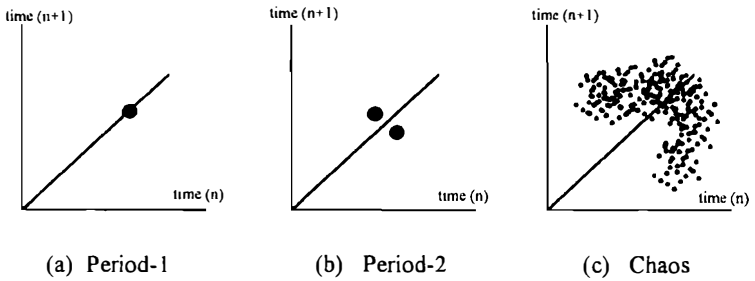
deal about the system dynamics. We learn in this experiment that we need not dismiss as intractable the analysis of a system that seems to be too large or has "too many variables" or "too many degrees of freedom." (One can surely imagine quite a few military systems with these imposing properties, starting with a conventional battlefield.) The water drops give us hope: by isolating and controlling *one key parameter* and making one straightforward measurement, we can still come to understand, and perhaps manipulate, a very complicated system.

The second common feature of Chaos illustrated by the dripping faucet is the presence of this key *control parameter*—in our case, the flow rate, controlled by the spigot. Think of a control parameter as a single knob that allows regulation of the amount of energy in the system. Not only does this energy control provide a means to dictate the dynamics of the dripping faucet, but the *transitions* between various behaviors are *identical* in countless, seemingly unrelated, physical systems. In the faucet, for instance, low flow generates periodic output; an increase in flow leads to higher-period behavior; even higher flow—more energy in the system—allows chaotic dynamics. Moreover, the Chaos appears when the system *has insufficient time to relax and recover* before the next "event" occurs. These same transitions take place in mechanical, electrical, optical, and chemical systems. Even more surprisingly, the transitions to more complicated behavior can occur at predictable parameter values ("knob" settings), a result that will be treated in the demonstration that follows.

The critical conclusion is that our knowledge of chaotic systems teaches us to expect specific behaviors in a system that displays periodic behavior; to expect to see higher periods and Chaos with more energy input; and to forecast, in some cases, parameter values that permit these transitions.

A third common characteristic of chaotic systems highlighted here is the fact that the system dynamics are revealed by observing *time intervals between events*. The physical event—droplet formation and break-off—is impossible to simulate, so we avoid taking difficult measurements like drop diameter, drop mass, or velocity. Instead, we note the length of time between events; if we can measure this accurately, we are able to construct a *return map* or *first-return map* that clearly indicates the various patterns of behavior (figure 6).

On the $x$-axis, a return map plots the time difference between, say, drops 1 and 2, versus the $y$-axis, which plots the time difference between the next two—here, drops 2 and 3. When the flow is slow and periodic, the time intervals are regular, so the time between the first drops is equal to the time between the next pair of drops. On the plot, that means we are plotting $x$-values and $y$-values that are always equal, so we see a single dot on the plot (figure 6a). So, if we ever observe a return map where all the data fall on a single point, we can conclude the system is behaving periodically.




(a) Period-1 (b) Period-2 (c) Chaos

*Figure 6. First-Return Maps for the Dripping Faucet.*

If we consider our time-difference measurement a record of the state of our system, then any *limit* behavior summarized on the return map represents an *attractor* for the system. An attractor is a collection of states that a system "settles" into after its transient dynamics die out. For the periodic flow, the attractor is a single point on the return map.

The next transition in drop dynamics was reported by Shaw and Scott but is fairly difficult to perceive in our home experiment. At a specific range of flow rates, before the onset of Chaos, they observed a rapid string of drops that fell off in close pairs. The flow showed a different periodicity, with one short time-step followed by a longer time-step: drip-drip drip-drip drip-drip. They confirmed the existence of this change in periodicity by using a simple model of their system, but its presence was clear on the return map (figure 6b).[12] In this case, we say the sequence of drops has *period-2*, that the system has undergone a *period doubling*, and that the attractor is the set of two points on the plot. For the record, this system (like many others) experiences additional period doublings to *period-4*, *period-8*, etc., before the onset of Chaos. These transitions, however, can be difficult to detect without sensitive laboratory equipment.

Finally, chaotic flow generates time intervals with no periodicity and no apparent pattern. However, the chaotic return map does not simply fill all the available space with a random smear of points. There is some rough boundary confining the chaotic points, even though they appear to fill the region in an erratic, unpredictable way (figure 6c). What is most astonishing is that this smear of points is amazingly reproducible. That is, we could run the experiment anywhere, with virtually any water source, and a very similar pattern of points

would appear on the return map for chaotic flow. The structure of the collection of points is due to the dynamics of water drops in general, not the specific experimental machinery.

The water drop experiment offers additional hope that we might *control* a chaotic system. (What follows is easiest to demonstrate if you use a portable water spout, like an empty mustard bottle, but it may work well if your kitchen spout is sufficiently flexible.) Set the spout so you have flow that remains chaotic. Then jiggle the spout in some regular, periodic way. You might bounce the mustard bottle up and down, or simply tap the end of your kitchen faucet with a regular beat. You should be able to find the right strength and frequency to perturb your system and get it to change from Chaos back to periodic drips, with a periodicity that will match the beat of your tapping. This is not very different from kicking your legs on the swing. However, in this case, we are *starting* with a chaotic system and applying a relatively small disturbance to *force* the system to return to more stable periodic behavior.

Later discussion will offer more details concerning Chaos control that has been demonstrated successfully in both theory and practice. We will also consider issues of when we might prefer Chaos to be present, or not present, in a given system. At this point, it is interesting to notice that the perturbation of the dripping faucet can drive a chaotic system into stable (periodic) behavior, while our previous perturbation of the park swing forced it to go from stable periodicity into Chaos.

So far we have introduced two chaotic systems whose dynamics will lend some insight to the behavior of more complicated military systems. The first was mechanical, the second fluid. Our next demonstration involves some simple (and inexpensive) electro-optics that can be picked up at any hardware store.

## Night-light

I stumbled onto this demonstration quite accidentally, when I went to plug in a small night-light in our bathroom—one of those automatic lights, about the size of your palm, that turns on automatically when the room is dark. I plugged it into the socket; the room was dark. Just before I pulled my hand away from the night-light, it flickered rapidly. I put my hand near the light again and I saw the same flicker. What interesting dynamics are hiding in this system?

What's the system? To reconstruct this system we need a light source of any kind that includes an automatic sensor that cuts off the electric current when it senses light (figure 7). We also need a dark room and a mirror, small enough so

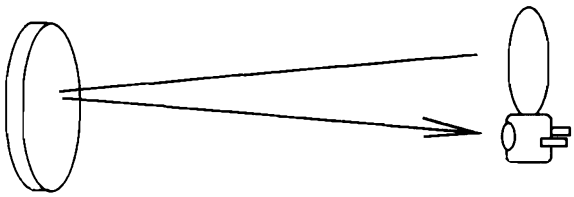


*Figure 7. Night-light with Feedback.*

we can move it around near the light, and supported in a stand so we can steady the mirror and observe the light. Now, set the mirror so it reflects light from the bulb back onto the sensor (as my palm had for my night-light in the bathroom). By adjusting the mirror's distance from the sensor, we can vary the delay of *feedback* in the system.

What are we observing and measuring? When the mirror is close enough to the night-light, about four to twelve inches, you should see it flicker. What's going on? Quite simply, the sensor is doing its best to fulfill its mission under unusual circumstances. Initially, the room is dark, so the sensor turns its light on; but with the mirror in place, as soon as the light turns on, the sensor picks up the reflected light and correctly decides to shut off. Oh dear, the sensor mutters, the room is dark again: time to turn on, and so on. The sensor detects and responds very quickly, so we see the night-light flicker vigorously.

What exactly should you observe in this system? Like the dripping faucet, the output to measure here is the *frequency*—in this case, the flickering frequency—the time difference between events. We would probably learn even more by also monitoring the light's intensity, but this requires fancier equipment than most of us keep around the house.

What transitions should we expect? To see the range of dynamics possible in this system, start with the mirror far from the sensor, about a yard or so away. Slowly draw the mirror closer to the sensor. The first change you'll see is a noticeable dimming in the light. Honestly, I don't know yet whether this is a simple change in the light's output or a fluctuation whose frequency exceeds our visual resolution. Do your best to locate the farthest point from the light where

the dimming begins. Let's label this distance $d_0$. You may find that $d_0$ is up to a foot or two away from the light.

As you move the mirror even closer, the next change you'll probably see is the first sign of flickering. Once again, try to mark the farthest place where the flicker is noticeable and label it $d_1$. As you continue to move the mirror toward the sensor, you will see various ranges of distances where the flickering displays other periodicities, and you ought to see at least one region where the reflected feedback drives the system into Chaos: irregular bursts of brightness and flickering. Mark the distances, as well as you're able, where you see transitions: $d_2$, $d_3$, etc. If you don't observe any Chaos, how might you alter your system? There are several accessible control variables: try a different (cleaner?) mirror; change your reflection angle (are you hitting the sensor efficiently?); or use a brighter light bulb.

What's the significance? The dynamics exhibited by the night-light system point to several critical insights that will help us apply the general results of Chaos theory to other systems. The first new insight comes from the dynamics we can generate by imposing *feedback* on a system. Of course, the use of feedback itself is not new, but the output we observe in the night-light provides a new understanding of the dynamics that control theorists have wrestled with for decades.

The night-light demonstration also offers practical new approaches to the study and control of a system whose output sometimes fluctuates. In particular, once I observed periodic behavior in the system (accidental though it was), I knew to expect several ranges of periodicity and Chaos if I varied one of the control parameters available to me. That is, my experience with Chaos gave me very specific behaviors to *expect*, in addition to obvious suggestions of ways to *control the dynamics*. Moreover, I had some idea of the kinds of dynamics to expect *without knowing anything* about the internal workings of the system!

This universality of chaotic dynamics underscores the power of understanding the basic results of Chaos theory. Certainly, not every system in the world is capable of generating Chaos, but in many systems we can control and analyze a system with no need for a model.

Here are two simple examples of the kind of analysis that is possible, even without a model. For this analysis we need only the list of distances ($d_0$, $d_1$, etc.) where we noted transitions in system behavior. First of all, we know that the signal in our demonstration, the light from the bulb, is traveling at a known constant, $c = 3.0 \times 10^8$ meters/second. Therefore, we can quickly assemble a list of important *time constants* for this system by dividing each of our distances by the speed of light, $c$. These time constants directly affect important transitions in the light's output; we know we can alter the system's behavior by applying

disturbances that are faster or slower than these key time delays. Other time constants we might consider include the frequency of the electric current and the frequency (color) of the light.

A second numerical result gives us some hope of predicting the parameter values where transitions in dynamics should occur. Dr. Mitchell Feigenbaum of Los Alamos National Laboratory, New Mexico, discovered that many chaotic systems undergo transitions at predictable ranges of their parameter settings. In particular, he compared the ratio of differences between key parameter values, which for us translates into calculating a simple ratio:

$$(d_0 - d_1) / (d_1 - d_2) \qquad (2)$$

He discovered that this ratio approaches a universal constant, approximately 4.67—now known as the Feigenbaum number—which appears in chaotic systems where Chaos arrives via period doubling, such as in our dripping faucet. This amazing result tells us when to anticipate changes in dynamics. For instance, if our first transition happens when the mirror is 12 inches out, and the second transition occurs at 8 inches, we note the difference in these parameter values, 4 inches (figure 8). Feigenbaum tells us that we ought to expect another transition ($d_1 - d_2$) in another 4/4.67 inches, or 0.85 inches from the point of the last transition.

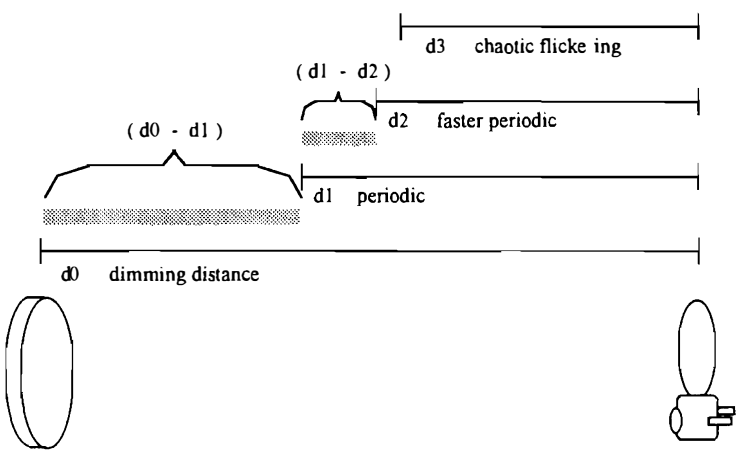



*Figure 8. Finding the Feigenbaum Constant in the Night-light Experiment.*

Now, in any system where we try to make predictions this way, we may face other complications. Our moving mirror, for example, may actually change several control parameters at once, such as brightness and focus. However, the mere existence of the Feigenbaum constant gives us hope of being able to anticipate critical changes in complicated systems; in fact you should find that this prediction works quite well for your measurements with your night-light system!

This third home demonstration brings to light several key results that apply to many chaotic systems. In particular, the demonstration illustrates: the potential dynamics that can be generated by imposing *feedback* on a system; very specific behaviors to *expect* in chaotic systems; suggestions of ways to *control* a system's dynamics; ways to analyze and control a system with *no need for a formula or model*; and how the Feigenbaum constant helps *anticipate system transitions.*

## Other Home Demonstrations

Many other systems you see every day exhibit chaotic dynamics. Watch the cream stir into your coffee. How does a stop sign wobble in a rough wind? Think about the position and speed of a car along a major city's beltway. What are the states of *all* the cars traveling the beltway?[13] Watch the loops and spins of a tire swing in a park. If you are really adventurous, hook up your home video camera as it shows a live picture on your television set, then aim the camera at the television set and zoom in and out to generate some exciting feedback loops.

Consider how you might carefully describe those systems. What can you observe and measure in those systems? What are the important parameters? As the control parameters increase or decrease, what transitions in behavior should you expect?

I have summarized several home demonstrations in this chapter to develop some intuitions and to introduce the vocabulary and tools of dynamical systems. I hope they spark your imagination about comparable systems that interest you. More importantly, they may represent your first understanding of chaotic systems, so you can begin to expect and anticipate Chaos in *your* systems. The next chapter adds more detail to the vocabulary and ideas introduced here.

# II

# Definitions, Tools, and Key Results

Of all the possible pathways of disorder,
nature favors just a few.[14]

James Gleick

The previous chapter described a few simple demonstrations so that we could begin to develop some basic intuition for chaotic dynamics. I also used some of the associated Chaos vocabulary in those demos in order to introduce the definitions in the context of real systems. Detailed definitions require too much time to present in full. However, we need to review some vocabulary with care, since the tools for observing and exploring complex dynamics are linked closely to the vocabulary we use to describe our observations. Rather than pore through excruciating details of precise definitions, this chapter concentrates on the *consequences* of the definitions. The focus will be to answer questions such as, "What does it *mean* to be an attractor?"

We will narrow the discussion to the most important issues: What *is* Chaos? How can we test for it? What does it mean to me if I have Chaos in my system? By concluding with a summary of Chaos theory's key results, the way will be paved for later chapters that suggest ways to apply those results.

This chapter concentrates on two classic chaotic systems: the logistic map and Lorenz's equations for fluid convection. These two examples reinforce some of

the lessons learned in the last chapter, and they make a nice bridge to the military systems examined in the next chapter. In particular, I will apply common Chaos tools to these two examples so that the reader can visualize the kind of new information Chaos theory can provide about a system's behavior.

## The Logistic Map

What is the system? Our first case looks at the work of biologist Robert May, who in the early 1970s researched the dynamics of animal populations. He developed a simple model that allowed for growth when a population of moths, for instance, was small; his model also limited population growth to account for cases of finite food supply.[15] His formula is known as the *logistic equation* or the *logistic map*.

What are we observing and measuring? The logistic map approximates the value of next year's population, $x[n+1]$, based on a simple quadratic formula that uses only information about this year's population, $x[n]$:

$$x[n+1] = \lambda\, x[n]\,(1 - x[n]).$$

The parameter $\lambda$ quantifies the population growth when $x[n]$ is small, and $\lambda$ takes on some fixed value between 0 and 4. In any year $n$, the population $x[n]$ is measured as a *fraction*, between 0 and 1, of the largest community possible in a given physical system. For example, how many fish could you cram into the cavity filled in by a given lake? The population $x[n]$ expresses a *percentage* of that absolute maximum number of fish.

It is not too hard to illustrate the dynamics of the logistic map on your home computer. Even with a spreadsheet program, you can choose a value for $\lambda$ and a starting value for $x[1]$, and calculate the formula for $x[2]$. Repeated applications of the formula indicate the changes in population for as many simulated years as you care to iterate. Some of the dynamics and transitions you should expect to see will be discussed in this chapter.

What's the significance? One helpful simplification of May's model was his approximation of continuously changing populations in terms of *discrete time intervals*. Imagine, for instance, a watch hand that jerks forward second by second instead of gliding continuously. Differential equations can describe processes that change smoothly over time, but differential equations can be hard to compute. Simpler equations, *difference equations*, like the logistic map, can be used for

processes that jump from state to state. In many processes, such as budget cycles and military force reductions, changes from year to year are often more important than changes on a continuum. As Gleick says, "A year-by-year facsimile produces no more than a shadow of a system's intricacies, but in many real applications the shadow gives all the information a scientist needs."[16]

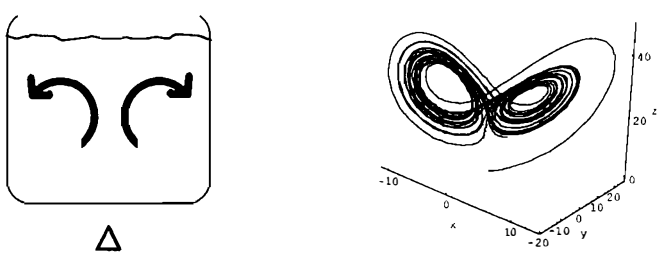
The additional beauty of the logistic map is its simplicity. The formula includes nothing worse than an $x^2$ term—how badly can this model behave? Very shortly, you will find that this simple difference equation produces *every* significant feature common to most chaotic systems.

## The Lorenz Equations

What's the system? Our second case began as a weather problem. Meteorologist Edward Lorenz wanted to develop a numerical model to improve weather predictions. Focusing on a more manageable laboratory system—the convection rolls generated in a glass of heated water—Lorenz modified a set of three fairly simple differential equations.[17]

$$\begin{aligned} x' &= -\sigma x + \sigma y \\ y' &= Rx - y - xz \\ z' &= -Bz + xy \end{aligned} \tag{3}$$

What are we observing and measuring? The phase variables, $x$, $y$ and $z$ combine measurements of the flow as the heated water rises, cools, and tumbles over itself (figure 9a). The $x$ variable, for instance, is proportional to the intensity




*(a) The System* *(b) A Trajectory in Phase Space*[18]

*Figure 9. Lorenz' Weather-in-a-Beaker.*

of the convection current; $y$ is proportional to the temperature difference between the rising and falling currents. The numbers $\sigma$, $R$ and $B$ are the system's physical parameters, which Lorenz set at $\sigma = 10$, $R = 28$, and $B = 8/3$. As the phase variables change in time, they trace out fascinating patterns, like those illustrated in figure 9b.

What's the significance? The Lorenz equations crudely model only one simple feature of fluid motion: temperature-induced convection rolls. However, even in this simple system, Lorenz observed extreme sensitivity to initial conditions as well as other symptoms of Chaos we will see momentarily. He clearly proved that our inability to predict long-term weather dynamics was *not* due to our lack of data. Rather, the unpredictably of fluid behavior was an immediate consequence of the *nonlinear rules* that govern its dynamics.

## Definitions

Now that we have two new systems to work with, along with the "experience" of our home demonstrations, let's highlight the vocabulary we will need to discuss more complicated systems.

*Dynamical System.* Recall how we defined a system as a collection of parts along with some recipe for how those parts move and change. We use the modifier *dynamical* to underscore our interest in the character of the motions and changes. In the case of the logistic map, for example, the system is simply a population measured at regular time intervals; the logistic equation specifies how this system changes in time.

*Linear and Nonlinear.* The adjective *linear* carries familiar geometrical connotations, contrasting the linear edge of a troop deployment, for example, with the curved edge of a beach. In mathematics, the concept of linearity takes on broader meaning to describe general *processes*. We need to understand linearity because *isolated linear systems cannot be chaotic.* Moreover, many published explanations of linearity make serious errors that may prevent you from grasping its significance.

Some authors condense the definition of linearity by explaining that in a linear system the output is proportional to the input. This approach may be helpful when we model the lethality of certain aircraft, saying that three sorties will produce three times the destruction of a single sortie. However, there is at least one further level of insight into linearity. That insight comes from our first home demonstration, the pendulum.

Even though a pendulum swings in a curve and we describe its motion with sine and cosine functions, an ideal pendulum is a *linear* system! It's linear because the equation that defines its motion has only linear operations: addition and multiplication by constants. Common *nonlinear* operations include exponents, trigonometric functions, and logarithms. The important consequence is that the solutions to most linear systems are completely known. This may not seem earth-shattering for a single pendulum, but many oscillating systems—such as vibrating aircraft wings, mooring buoys, and concrete structures subjected to shock waves—behave just like a collection of coupled pendulums. Therefore, as long as they aren't regularly "kicked" by external forces, those real systems are just enormous linear systems whose range of possible motions is completely known.

Without delving into the subtleties of more analytical definitions, here are some important consequences of the property of linearity:

- The solutions to linear systems are known (exponential growth, decay, or regular oscillations), so linear systems *cannot* be chaotic.
- "Kicking" or forcing an otherwise linear system *can* suffice to drive it into Chaos.
- If Chaos appears in a system, there *must* be some underlying nonlinear process.

This discussion of linearity should serve as a wake-up call. Basically, if you have a system more complicated than a pendulum, or if you see an equation with nonlinear terms, you should be alert for possible transitions from stable behavior to Chaos. This is certainly a simplification, since many systems include additional control mechanisms that stabilize their dynamics, such as feedback loops in human muscles or in aircraft control surfaces. However, the minimum ingredients that make Chaos possible are usually present in systems like these. In the absence of any reliable control, unpredictable dynamics are not difficult to generate.

*Phase Space and Trajectories.* A system consists of components and their rules of motion. To analyze a system one must decide exactly what *properties* of those components to measure and record. The time-dependent properties necessary to determine the system dynamics are known as the system's *phase variables*. The collection of all possible combinations of values those variables can attain is then the *phase space* for our system.

Phase space is the canvas where a system's dynamics are painted. The Lorenz equations, for example, define the time-dependent changes of fluid flow in a

heated beaker of water. If we start at some initial state and let the system evolve in time, we can track how the three system variables change. We can then plot that information with a three-dimensional curve (figure 9b). Notice that the curve does not directly illustrate the physical motion of the water. Rather, the curve indicates changes in all three phase variables; at least one of these—the temperature gradient, $y$—quantifies changes we cannot see. The plot's entire three-dimensional space constitutes the phase space for the Lorenz equations; we call the single curve that leaves a particular initial state a *trajectory* (or *trajectory in phase space*) for that initial condition.

*Parameter.* A *parameter* is a quantity that appears as a constant in the system's equations of motion. The logistic map has only one parameter, $\lambda$, which expresses the rate of growth for small populations. A pendulum's parameters include its mass and the length of its bar. Sometimes a parameter expresses a *physical constant* in the system, such as the gravitational constant for the pendulum. Most important, a system parameter often represents a control knob, a mechanism to control the amount of energy *in a system.*

For instance, we saw earlier how changes in flow rate, the key parameter for the dripping faucet, drove transitions in system output. In the following section on Chaos Tools, we'll see how the logistic map undergoes transitions as we increase $\lambda$ from 0 to 4. It is important to note that even in relatively simple systems like the faucet, there are many influential parameters that are not easily controlled: spout diameter, mineral content of the water, local humidity, spout viscosity, etc. One crucial skill for any decision maker is the ability to identify all the parameters *accessible* to external control, and to *isolate* those parameters that have the greatest influence on a system.

*Sensitivity to Initial Conditions (SIC).* Any system "released" from its initial state will follow its laws of motion and trace some trajectory in phase space, as we saw with the logistic map above. However, if a system is sensitive to initial conditions we also know that *any two* initial states that deviate by the slightest amount *must* follow trajectories that diverge from each other exponentially. Consider figure 10. The lower series started from an initial population only slightly greater than the upper series; after about sixteen iterations, the two trajectories bear no resemblance to each other. This is an illustration of SIC.

We can measure *how fast* neighboring trajectories tend to diverge. At any given point in phase space, a *Lyapunov* (lee-OP-uh-noff) *exponent* quantifies this rate of divergence. This exponent has properties that come from its role as the constant $k$ in the exponential function $e^{kt}$. If $k$ is negative, then small disturbances tend to

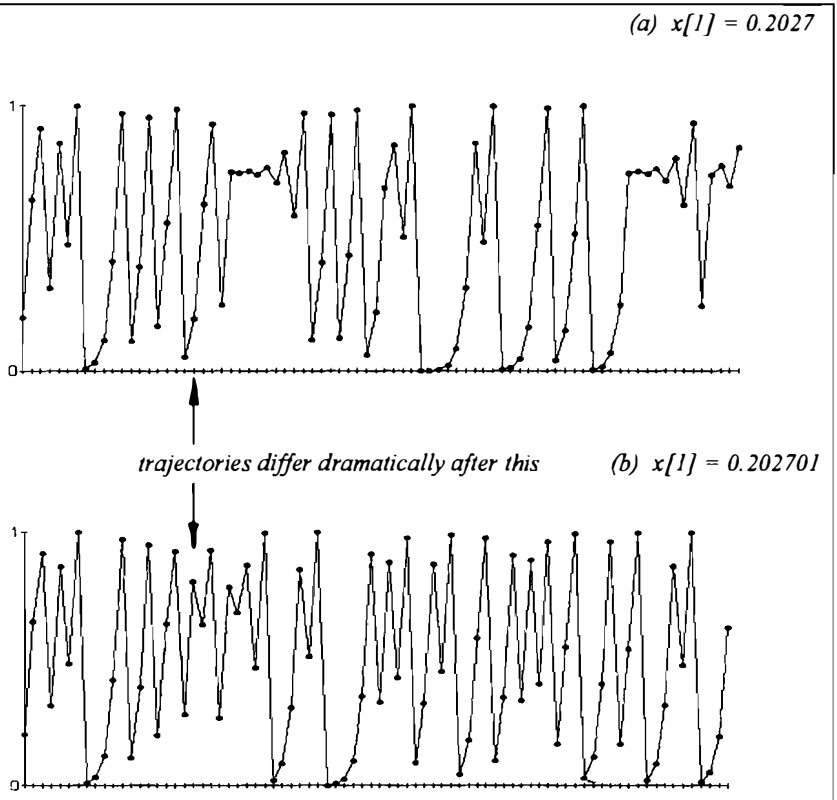



*Figure 10. Chaotic Trajectories of the Iterated Logistic Map.*[19]

get smaller, indicating no SIC; if $k$ is positive, small perturbations increase exponentially. With these measurements, we can estimate how "touchy" a system is, how *vulnerable* the system may be to external disturbances, and *how unpredictable* the consequences of our actions may be. We can often calculate an *average Lyapunov exponent* for an entire region of phase space. This allows us to compare two systems, or two scenarios, and decide which one tends to be more or less predictable. Information like this could prove valuable for prioritizing the courses of action available to a commander.

Many systems, as we say, are SIC, including some non-chaotic systems. For example, take the simplest case of exponential growth, where a population at any time $t$ is given by a recipe such as $e^{3t}$. This system is SIC, but certainly not

chaotic. What does this mean for us? If a system is SIC, you are not *guaranteed* to find Chaos; if, however, a system is *not* SIC, it *cannot* exhibit Chaos. Thus we have identified SIC as a *necessary but not sufficient* condition for Chaos to occur.

*Attractors.* Despite the fact that chaotic systems are SIC, and neighboring trajectories "repel" each other, those trajectories still confine themselves to some limited region of phase space. This *bounded* region will have maximum and minimum parameter values beyond which the trajectories will not wander, unless perturbed. In the logistic equation, the population remains bounded between the values of 0 and 1, though it seems to take on every possible value in between when it behaves chaotically.

In the Lorenz equations, the trajectories also stay within finite bounds, but the trajectories do not cover all the possible values within those limits. Instead, a single trajectory tends to trace out a complicated, woven surface that folds over itself in a bounded region of phase space (refer to figure 9). The collection of points on that surface is an *attractor* for those dynamics; the classic Lorenz attractor is a particularly striking example.

Left to itself, a single trajectory will always return to revisit every portion of its attractor, unless the trajectory is perturbed. All chaotic, or *strange*, attractors have this *mixing property*, where trajectories repeatedly pass near every point on the attractor. Envision where a single droplet of cream goes after it is poured into coffee.[20] Or imagine the path of a single speck of flour as it is kneaded into a ball of dough. If the mixing continued long enough, the small particle could be expected to traverse every neighborhood of its space. Actually, one way to sketch a rough image of an attractor is simply to plot a single trajectory in phase space for a very long time.

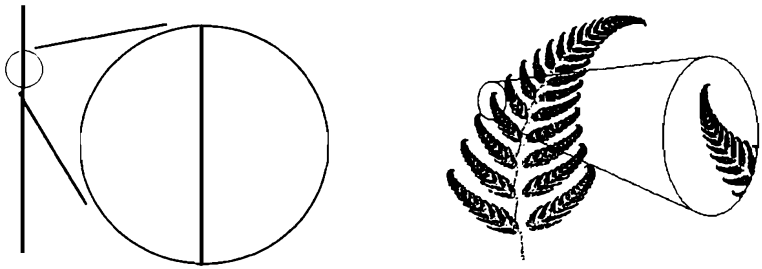
Transient states are all the initial conditions off the attractor that are *never* revisited by a trajectory. If we gather together all the transient states that eventually evolve toward a single attractor, we define the *basin of attraction* for that attractor. Thus, the basin represents all the possible initial states that ultimately exhibit the same *limit dynamics* on the attractor. In the Lorenz system, for instance, we might start the system with a complicated temperature distribution by dropping an ice cube into hot water. However, that transient extreme will die out, and after a while the system must settle down onto the collection of temperature variations that stay on the attractor. Because of SIC, the precise state of the Lorenz system at any given time cannot be predicted. However, because the attractor draws dynamics toward itself, we *do* know what the *trends* in the dynamics have to be!

When those trends are examined closely, a single trajectory will be found to visit certain regions of the attractor *more often* than others. That is, if we color each point on the attractor based on how often the trajectory passes nearby, we will paint a richly detailed *distribution* of behavior on the attractor. To picture this, visualize the distribution of cars on the interstate beltway around a big city. At any time of a given day, we could note the number of vehicles per mile and begin to identify patterns of higher traffic density for certain times of day. We could continue and consider the distribution of cars on whatever scale interests us: all interstates, all streets, or just side streets. Even though we cannot predict the number of cars present on any particular street, these distributions and patterns give us crucial information on how the overall system *tends* to behave.

The properties of attractors are key signposts at the junction where Chaos theory matures past a mere metaphor and offers opportunities for practical applications. Attractors provide much more information than standard statistical observations. This is because an attractor shows not only distributions of system states but also indicates "directional" information, that is, how the system tends to change from its current state. As a result, when we construct an attractor we reconstruct *an image of the system's global dynamics*—without appealing to any model. In subsequent chapters, we will show how this reconstruction allows us to *predict short-term trajectories* and *long-term trends*, to perform *pattern recognition*, and to carry out *sensitivity analysis* to help us make strategic decisions.

*Fractal.* Though there are standard definitions of several types of *fractals*, the important consequence for us is that fractals describe the complexity, or the amount of detail, present in objects or data sets. A well-defined line, like the $y$-axis on a graph, is *one-dimensional* because one piece of information, the $y$-coordinate, suffices to pinpoint any position on the line. To get an idea of what *dimension* means in a fractal sense, first imagine using a microscope to zoom in on an ideal line. However intently we zoom in, the most detail we can expect to see is a razor-thin line cutting across the field of view (figure 11a). If, as a second case, we focus the microscope on a *two-dimensional object*, like a square, sooner or later the narrow field of view will fill with an opaque image. We need *two coordinates* to pinpoint any place on that image.

On the other hand, a *fractal image* has a non-integer dimension. An image with dimension 1.7, for instance, has more detail than a line but too many holes to be worthy of the title two-dimensional. Fractal images contain infinite detail when we zoom in (figure 11b). The good news is that the extraordinary detail present in fractal images can be generated by very simple recipes.



*(a) A line is just a line.* *(b) Ferns within ferns.*[21]

*Figure 11. Fractal Dimension: Always More Detail When You Zoom.*

The term "fractal" refers specifically to a mathematical dimension defined by executing this zooming process very precisely. First, assume the line in figure 11 is a centimeter (cm) long. It only takes one circle of 1 cm diameter to completely cover the line. If I cover it with circles 1/2 cm across, I need two. Similarly, I need 17 covering circles of diameter 1/17 cm, or 1986 circles of diameter 1/1986. Since the number of circles needed to cover the image scales is (1/diameter) to the *first* power, we say that image has dimension "1." This comes as some relief, since we all survived geometry class knowing that lines are one-dimensional.

Now consider the complex fern in figure 11. If its total length is about 1 cm, a single large circle will cover it. However, as we start to cover it with smaller and smaller circles, we find that we need fewer circles than we would need if we were trying to cover a solid square (of dimension 2). In fact, the number of circles needed scales like (1/diameter) raised to the 1.7 power. We say, then, that the fern has dimension 1.7, and since that dimension is not an integer, or fractional, we call the image a "fractal."

The study of fractal geometry becomes important to military applications of Chaos in three main areas: image compression, dimension calculation, and basin boundaries. In *image compression*, the infinite detail generated by simple sets of fractal instructions allows mathematical *instructions* rather than pixel-by-pixel values to be transmitted; the image can then be recreated by the receiver using the instructions.

The second application, *dimension calculation*, is possible with time series as well as with geometric figures; when we calculate the dimension of a sequence of data points, we get an estimate of the *minimum number of variables* needed to model

the system from which we measured the data. Often the estimate lies very close to the number of variables needed in a model, thus saving analysts the struggle of developing overly complex situations.

Thirdly, many systems that have two or more attractors also have two or more basins of attraction. Very often, the *boundaries* between basins are not smooth lines. Instead, the basins overlap in fractal regions where one initial condition may lead to steady state behavior, but *any* nearby initial condition could lead to completely different behavior. Consider the illustration in figure 12, the basins of attraction for a numerical model. All the initial conditions (white areas) lead to one kind of behavior; all the dark points lead to entirely different behavior. A commander making decisions in such an environment will have to be alert—small parameter changes in certain regions produce dramatic differences in outcomes.[22] For instance, the pictured decision space might simulate, on one

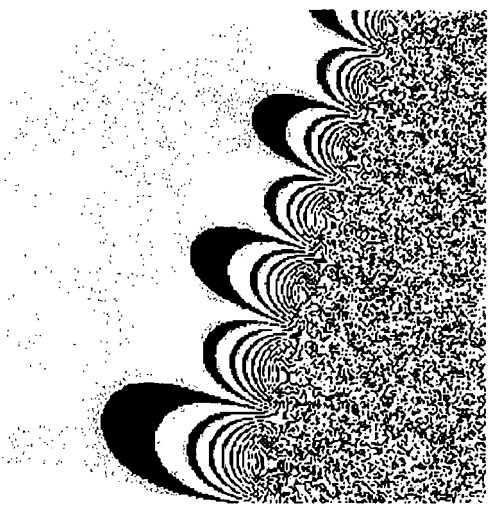


*Figure 12. Fractal Boundaries Between Basins of Attraction.*[23]

axis, the number of troops available for reinforcement, while the other axis indicates time intervals between sending in fresh troops. If the combat simulation indicates eventual victory with a black dot, and defeat with white, commanders would need to choose reinforcement strategies with great care in order to turn the scenario's outcome in their favor.

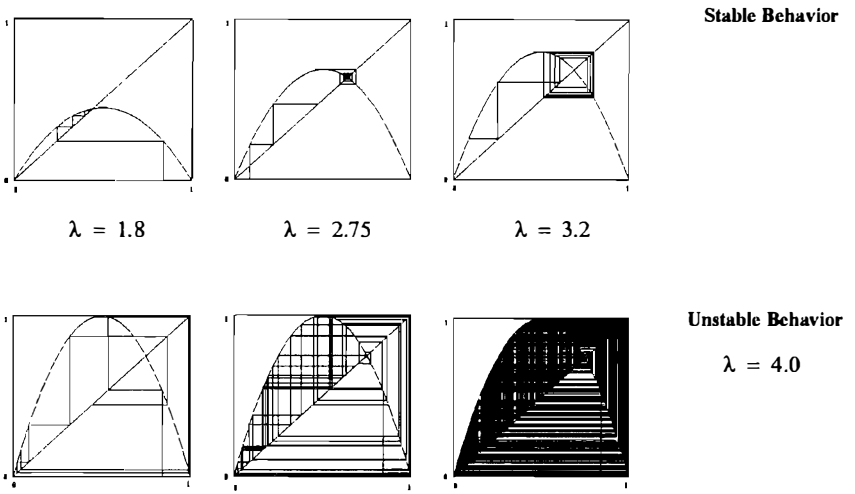
*Bifurcation. Bifurcation theory* represents an entire subdiscipline in the study of dynamical systems. I mention bifurcations here for two reasons. First, so you will recognize the word in other references. In the context of the demonstrations thus far, a bifurcation is simply a *transition in dynamics*. The faucet, for example, drips slowly when the flow rate is low. At some higher flow rate, the drops come out with period-2; we say the system has undergone a bifurcation from one kind of periodicity to another. A bifurcation is a transition in system dynamics due to a change in a control parameter.

The second reason for offering this new terminology is to highlight the *universality* of bifurcation types. That is, when one system parameter is changed, you may see subtle bifurcations or catastrophic ones, but a few *classes* of bifurcations are common to many dynamical systems.[24] Recall the discussion of transitions in the night-light demonstration. The transitions came at smaller and smaller intervals, roughly according to patterns predicted by the Feigenbaum constant (refer to figure 8). Feigenbaum first discovered this constant through his study of the logistic map, where transitions occur in the same pattern as in the night-light. Overall, the most important consequence of Feigenbaum's discovery is that the same *transitions* he observed in the logistic map also appear in many diverse physical systems.

*Dense, Unstable, Periodic Orbits.* Consider one last feature of the logistic map that ultimately makes it possible for us to *control* chaotic systems. Chaos control will be addressed in the next chapter; for now, we take a few steps through the dynamics of the logistic map in order to glimpse the complicated activity on an attractor, as illustrated in figure 13.

Suppose we set the parameter to a small value, say $\lambda = 1.8$. We can start the system with $x[1]$ anywhere between 0 and 1, and successive iterations of the logistic equation will always drive the value of $x[n]$ toward 0.44, a stable, fixed point. If we increase $\lambda$ to 2.75, the system still has a stable, fixed point, but that point is now around 2/3. Raising the control parameter produces no qualitative change in behavior. However, if we raise $\lambda$ slightly above 3, the system does *not* settle into a fixed point but falls into a cycle of period-2. Thus, at $\lambda = 3$ we see a bifurcation from stable to periodic behavior.

Transitions come hand-in-hand with *changes in stability*. Any system might have both stable and unstable behaviors. For instance, the equations governing a pencil standing on its point have a good *theoretical* equilibrium one with the center of gravity directly above the point—but we cannot stand a pencil on its point, because that state is unstable. That is, the slightest perturbation draws the system away from that state. On the other hand, a marble lying at the bottom of




*The parabolas trace out the logistic equation, $x[n] = \lambda x[n] (1-x[n])$.*
*The diagonal line holds the current population value, $x[n]$, until we iterate again, drawing a new vertical line to the parabola.*
*These "web diagrams" illustrate the long-term behavior of iterates.*

*Figure 13. Graphical Iteration of the Logistic Map.*[25]

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a bowl stays there, because if the marble is perturbed slightly in any direction, it just rolls back.[26]

The important feature for us hides in the chaotic trajectory "smeared out" in figure 13, when $\lambda = 4$. Inside that smear—the attractor for this chaotic system—many periodic cycles still exist; on paper, that is. The fixed point, for instance, still lives at the place on the graph where the parabola intersects the diagonal. However, that point is unstable, so a trajectory can never approach it. Similarly, we can calculate trajectories of period-2, period-3, every possible period. In fact, there are *infinitely many* unstable, periodic trajectories woven through the attractor, woven thickly in a way mathematicians call *dense*. That means that every area surrounding *every point* on the attractor is crowded with these "repelling," unstable, periodic trajectories.

So, on one hand, it is not useful to locate any of these periodic behaviors, because *all* these trajectories are unstable. On the other hand, recent experiments have demonstrated ways to force the system to follow one of these periodic behaviors. This is the power of *Chaos control*; as we will see later, the density of these trajectories is the property that makes this control possible.

## So How Do We Define Chaos?

A chaotic system MUST be:

- bounded;
- nonlinear;
- non-periodic;
- sensitive to small disturbances; and,
- mixing.

This is, perhaps, not so much a definition as it is a list of necessary ingredients for Chaos in a system. That is, without any one of these properties, a system cannot be chaotic. I believe my list is also *sufficient*; therefore, if a system has *all* these properties, it can be *driven* into Chaos.

Also, a chaotic system *usually* has the following observable features:

- transient and limit dynamics;
- parameters (control knobs);
- definite transitions to and from chaotic behavior; and
- attractors (often with fractal dimensions).

What is the significance of these properties? Measurements of transient and limit dynamics in a system provide *new information* not available to us before the advent of Chaos theory. Our comprehension of the role of parameters in system dynamics offers opportunities for *new courses of action*, to be described in subsequent chapters. Finally, the common properties of system transitions and attractors suggest *new expectations* of system behavior, as well as *new strategies* for coping with those expectations. For other, more detailed characteristics of chaotic data—such as exponentially decaying correlation and broad power spectra—you can refer to any one of the texts described in chapter V, "Suggestions for Further Reading."

*Random.* You may look at the above definition of Chaos and wonder if the processes we call "random" have those same properties. For those interested in

more detail, a discussion of one definition of "random" appears in the appendix. However, I will pause here to focus on one difference between *random* and *chaotic* dynamics. Please be aware that we are ignoring some large issues debated by Chaos analysts. Some argue, for instance, that the kind of dynamics we now call "random"—like a roulette wheel—simply come from chaotic systems, with no random variables, where we just do not know the model. In other cases, "noise," or random imperfections in our measurements—like radio static—may come from Chaos that happens on a scale we have not yet detected. *For our purposes, the primary feature distinguishing chaotic from random behavior is the presence of an attractor that outlines the dynamics towards which a system will evolve.*[27] Existence of such an attractor gives us hope that these dynamics are *repeatable.*

In the water-drop experiment, for example, if results were random, the experiment would not be repeatable. However, if you and I both run this test and I list my experimental parameters for you, such as nozzle diameter and flow rate, the key features of this system's dynamics will be replicated precisely by our two separate systems. Slow flow is *always* periodic. The system undergoes period doubling (period-2, then period-4, . . .) on the way to Chaos, as we increase the flow rate. Most important, for high flow rates, your chaotic return map for time differences between drops will produce a smear of points nearly identical to mine. If the system were exhibiting random behavior, these global features would *not* be reproducible.

## The Chaos "Con"

Before leaving this review of basic Chaos vocabulary, we need to examine the common mistakes and misinterpretations that appear in many papers on the subject. The sum of these errors constitutes the Chaos "con," the unfortunate collection of misleading publications that tend to crop up when writers investigate new topics. The con may come from well-intentioned authors who are new to the subject but miss some key concepts because they are constrained by time. Other cons may come in contract proposals from cash-starved analysis groups who might try to dazzle their readers with the sheer volume of their Chaos vocabulary. It is very important to avoid the con, both innocent and intentional, but most of all, don't con *yourself* by making any of the following common errors.

*"Chaos is too difficult for you."* Don't let anyone fool you: if you finished college, you can follow the basics of Chaos. Be suspicious of anyone who tries to tell you that the *general* concepts are beyond your grasp. Some authors will disguise this false claim with subtle references to the "mysteries of Chaos" or "mathematical

alchemy" or other vocabulary designed to intimidate their readers. Don't believe it, and don't pay these folks to teach you Chaos. You *can* learn it—just remember to take your time.

*"Linear is. . . ."* Remember that some writers will oversimplify the definition of linearity by waving their pen quickly at some phrase like "output is proportional to input." That comment is true only if a system's output and input are very carefully defined. Never forget that pendulums, swings, and springs are all linear systems! Make sure the author's definition of linearity admits these three important physical systems.

*Bifurcation.* What exactly bifurcates? Trajectories *don't* bifurcate, as some authors have claimed. A single trajectory can do only one thing. We may have a limited capacity to *predict* that behavior, but—as a light bulb can be only on or off at any fixed time—a single system can evolve through only one state at a time. Remember that a bifurcation is a qualitative change in system behavior that we observe as we change parameter settings. The bifurcation, or branching, takes place on plots of *parameter* values.

*"Complicated systems must be chaotic."* The fact that a system is complicated or has many components does not necessarily mean that it allows Chaos. For instance, many large systems behave like coupled masses and springs, whose linear equations of motion are completely predictable. Indeed, an old-fashioned clock is extremely complicated—but its very essence is to be predictable. Similarly, other large systems include reliable control mechanisms that damp out perturbations and do not permit sensitive responses to disturbances. Such systems do not exhibit Chaos.

*"We need many variables for Chaos."* Many of the same authors who claim that big systems must be chaotic also propagate the fallacy that simple systems cannot exhibit Chaos. Nothing could be further from the truth. In fact, the power of Chaos theory is that the simplest interactions can generate dynamics of profound complexity. Case in point: the logistic map produces every symptom of Chaos described in this paper.

*"Butterflies cause hurricanes."* When Edward Lorenz presented his findings of SIC in weather systems, he described *The Butterfly Effect*, the idea that the flapping wings of a butterfly in one city will eventually influence the weather patterns in other cities. This phenomenon is a necessary consequence of the sensitivity of

fluid systems to small disturbances. However, the butterfly effect often gets fuzzy in the translation. Be wary of authors who suggest that a butterfly's flap in California will become *amplified* somehow until it spawns a hurricane in Florida. Believe it or not, several often-cited reports make this ridiculous claim. Make no mistake, if a weather system has enough energy to produce a hurricane, then the storm's path will be influenced by butterfly aerodynamics across the globe. However, the system does *not* amplify small fluid dynamics; rather, it amplifies our inability to *predict* the future of an individual trajectory in phase space.

*"Chaos" versus "chaos."* One of the first signals of a weak article is when the author inconsistently mixes comments on mathematical Chaos and social chaos. Unless we can distinguish between the two, we cannot get past the metaphors of Chaos to practical applications. As will be explained below, the existence of Chaos brings guarantees and expectations of specific phenomena: attractors; complex behavior from simple interactions; bounded, mixing dynamics; and universal transitions—from stable to erratic behavior—that make Chaos control possible.

The worst consequence of the Chaos con is that the well-intentioned reader may not discern the important results of Chaos theory. These results highlight the common characteristics of chaotic dynamics, a useful template for the kinds of dynamics and applications we should expect in a chaotic system. A review of the most important results follows here; a discussion of their applications constitutes the remaining portion of this essay.

## Tools of Chaos Analysts

One of the most important outcomes of the study of Chaos theory is the extraordinary array of tools that researchers have developed in order to observe the behavior of nonlinear systems. I cannot emphasize enough that these tools are not designed solely for *simulated systems*. We can calculate the same information from experimental time series measurements when there is no model available, and often when we can measure only one variable in a multi-variable system! Moreover, decision makers need the skills to *differentiate* random behavior and Chaos, because the tools that allow us to understand, predict, and control chaotic dynamics have no counterpart in random systems.

For the military decision maker who can use these tools, two issues stand out:

*What are the preferred tests for deciding if a system is chaotic?*
*How can we tell the difference between randomness and Chaos?*

The analytical tools used by Chaos analysts answer these questions, among many others. Our brief summary of the most basic tools begins with an important reminder. We always need to begin our analysis by answering two questions: what is the system, and what are we measuring? For example, recall the dripping faucet system, where we observe the dynamics not by measuring the drops themselves but by measuring time intervals between events. Only after we answer those two questions should we move on to consider some of the qualitative features of the system dynamics:

- What are the *parameters*? Can we control their magnitude?
- Does the system perform many *repetitions* of its events?
- Are there inherent *nonlinearities* or sources of feedback?
- Does the phase space appear to be *bounded*? Can we prove it?
- Do we observe *mixing* of the phase variables?

When we have a good grasp of the general features of a system, we can begin to make some measurements of what we observe. We should note, however, that our aim is not merely to passively record data emitted from an isolated system. Very often our interest lies in *controlling* that system. In an article on his analysis of brain activity, Paul Rapp summarizes:

> Quantitative measures [of dynamical systems] assay different aspects of behavior, and they have different strengths and weaknesses. A common element of all of them, however, is an attempt to use mathematics to reconstruct the system generating the observed signal. This contrasts with the classical procedures of signal analysis that focus exclusively on the signal itself.[28]

Therefore, keep in mind that the tools presented here are not used for observation only. They provide the means to *re-create a system's rules of motion*, to *predict that motion* over short time scales, and to *control that motion*.

*Depicting Data.* We have already encountered most of the basic tools used for observing dynamical systems. The two simplest tools—*time series plots* and *phase diagrams*—display raw data to give a qualitative picture of the data's bounds and trends. A time series plot graphs a sequential string of values for one selected phase variable, as in the plot of population variation for the logistic map in figure 10. Sequential graphs give us some intuition for long-term trends in the data and for the system's general tendency to behave periodically or erratically.

Phase diagrams trace the dynamics of several phase variables at the same time, as the Lorenz attractor does in figure 9. The first piece of information apparent

from a good diagram is the nature of the system's attractor. The attractor precisely characterizes long-term trends in system behavior—how long the system spends in any particular state. This information translates directly into probabilities.

*Attractors and Probabilities.* As a demonstration of translating attractor dynamics into probabilities, consider the chaotic trajectories of the logistic map shown in figure 13. The smear of trajectories makes it obvious that the population *x[n]* takes on most of the values between 0 and 1; but is the smear of values *evenly distributed* across that range? One way to find out is to build a quick histogram: divide the interval from 0 to 1 evenly into a thousand subintervals; keep a count of every time the evolving population *x[n]* visits each subinterval. Figure 14 shows the results of such a calculation; we see from the figure that the trajectory of the logistic equation spends more time closer to 0 and 1 than it does to other values. To illustrate, if this equation modeled the number of troops assigned to a certain outpost, a distribution like this would tell a commander that the site tends to be fully staffed or nearly vacant, with noticeably less probability of other incremental options.

Probability information like this has several immediate uses. First, of course, are the probability estimates that commanders require to prioritize diverse

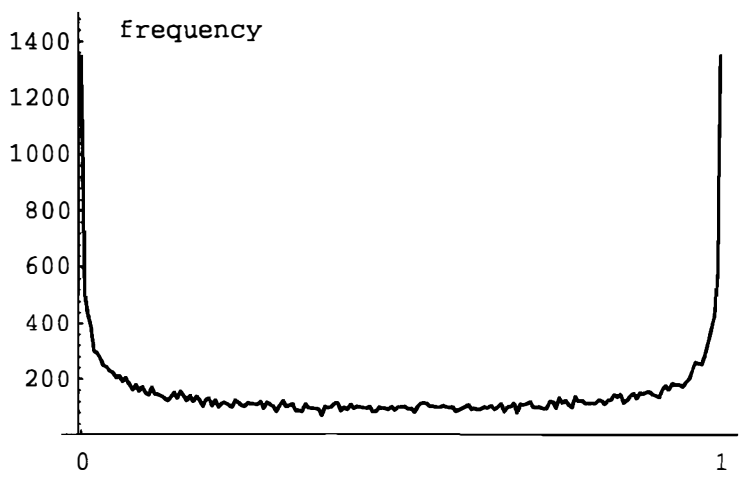


*Figure 14. Distribution of Logistic Map Dynamics.*[29]

courses of action. Second, analysts can use this information to compare models with real systems, to gauge how well the distribution of a simulated system relates to real data. Third, since many simple chaotic models use non-random formulas to generate distributions of behavior, the resulting distributions can be used in various simulations to replace black-box random number generators. We will explore these applications in greater detail in chapter IV.

*Attractors and Sensitivity.* As a single trajectory weaves its way through its attractor, we can also calculate local Lyapunov exponents (see pages 30-31) at the individual points on the attractor as well as an average Lyapunov exponent for the entire system. This exponent measures *how sensitive* trajectories are to small disturbances. Therefore, details about these exponents can guide decision makers to particular states where a system is more or less vulnerable to perturbation. The same exponents can also be calculated for various ranges of parameter settings so that commanders can discern which variables under their control may produce more predictable (or unpredictable) near-term outcomes.

*Embedding.* However directly we might calculate system features like attractors and Lyapunov exponents, how can we apply these tools to a real system where we have no descriptive model? Suppose we have a complicated system—like the dripping faucet—that gives us a time series with only one variable. What can we do?

The answer comes from a powerful technique known as *embedding*. Very simply, we can start with a sequence of numbers in a time series, and, instead of isolating them as individual pieces of data, we can group them in pairs. The resulting list of pairs is a list of *vectors* that we can plot on a two-dimensional graph. We can also start over and package the data in groups of three, creating a list of vectors we can plot in three-dimensions, and so on. This process *embeds* a time series in higher dimensions and allows us to calculate all the features of the underlying dynamics from a single time series. The suggested reading list in chapter V offers several sources that discuss this technique in detail.

Embedding is a powerful instrument for measurement because by embedding a time series we can calculate the *fractal dimension* of a data set. Since random data have theoretically infinite dimensions, and many chaotic systems have smaller dimensions, this is one of the first tools that can help us distinguish noise from Chaos.

Even more important, the dimension of a time series measures the amount of detail in the underlying dynamics and actually estimates the number of independent variables driving the system. So, when Tagarev measures a fractal

dimension of 2.9 for a time series of aircraft sorties (see figure 2), he presents strong evidence that the underlying system is *not* random but that it may be driven by as few as three key independent variables.[30]

Recent studies of embedded time series also have uncovered ways to use embedding as a vast, generalized grid through which we can interpolate to approximate a system's dynamics. In this way, researchers have made tremendous strides in predicting the short-term behavior of chaotic systems. More details of these results will be discussed in chapter IV.

*And Much, Much More. . . .* These tools represent only a small sample of the standard analytical tools currently in use. Consult the references highlighted in chapter V to find complete discussions of these and other tools, such as return maps, Poincaré sections, correlations, Fast Fourier Transforms, and entropy calculations. These tools constitute the primary sources of the *new information* that Chaos theory brings to decision makers.

## Results of Chaos Theory

Let us gather together the theoretical results scattered through these first two chapters. First, I will summarize the common features of chaotic systems. Then, I will review what it *means* for us to have Chaos in our systems.

Here is a brief snapshot of the common characteristics of Chaos, a sample of what to expect in a chaotic system. Most of these characteristics have been highlighted in our earlier examples. Not much is needed in a system in order for Chaos to be possible. In most physical systems, whose smooth changes in time can be described by differential equations, all that is needed are three or more independent variables and some nonlinear interaction. In difference equations, like the logistic map, where change occurs at discrete time intervals, all that is required is a nonlinear interaction.

Most systems have *accessible parameters*, system inputs we can control to adjust the amount of energy in the system. We should expect systems to have qualitatively different behaviors over different parameter ranges.

Surprisingly *common transitions*, from stable equilibria to periodicity and Chaos, occur in completely unrelated systems.

Influential dynamics occur on many different *scales*. For instance, the cloud cover that concerns forces during a combat operation is affected by the activity of butterflies across the globe. To understand the larger scale dynamics, we may need to consider the smaller scales.

*Attractors draw trajectories* towards themselves. So, if an attractor exists (in an isolated system), and the state of a system is in that attractor's basin, the system

cannot avoid proceeding toward the attractor. Dynamics on the attractor represent *global trends* of the underlying system, and they set global bounds on system behavior. The density of trajectories on the attractor also reveals the relative *distribution of behavior*.

Because of the trajectory mixing that takes place on attractors, the attractors are immersed in dense weavings of unstable periodic trajectories. The presence of these potential periodic behaviors makes *Chaos control possible*.

The universal nature of these properties helps us answer a somewhat bigger question:

## What does it mean to me to have Chaos in my system?

One consequence of understanding the results of Chaos theory is that if we are confident that a system can behave chaotically, then we *know* that it *must* have all the properties of Chaos. Some of these properties are hard to prove, but we "get them for free" if we know the system is chaotic. In particular, if a system is known to be chaotic, then we know, for example, that any models of that system *must* include nonlinear terms. We also know we have avenues to *control* the system; that is, any attractor for that system is densely woven with unstable periodic trajectories toward which we can drive the system (see the discussion of Chaos control in chapter IV).

In a 1989 Los Alamos report, David Campbell and Gottfried Mayer-Kress summarized *their* "lessons of nonlinearity":

1. Expect that nonlinear systems will exhibit *bifurcations* so that small changes in parameters can lead to qualitative transitions to new types of solutions.
2. Apparently random behavior in some nonlinear systems can in fact be described by deterministic *non-random* chaos.
3. Typical nonlinear systems have *multiple basins of attraction*, and the *boundaries* between different basins can have incredibly complicated *fractal* forms.
4. Our heightened awareness of the *limits to what we can know* may lead to more care and restraint in confronting complex social issues.
5. The *universality* of certain nonlinear phenomena implies that we may hope to understand many disparate systems in terms of new simple paradigms and models.
6. The fact that Chaos follows from well-defined dynamics with no random influences means that in principle *one can predict short-term behavior*.

7. The dense paths of trajectories on an attractor make *Chaos control* possible.[31]

To this list I would add that a basic understanding of Chaos brings not only limits to what we can know, but also new information about the dynamics that are possible. In the next chapter I outline some common military systems where one can expect to see Chaos. Then, in chapter IV we will be ready to learn how to apply these results.

# *Part Two*

## Who Needs Chaos Theory?

### *Applications*

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Big whorls have little whorls
Which feed on their velocity,
And little whorls have lesser whorls
And so on to viscosity.

Lewis F. Richardson[32]

Thank heaven
For little whorls.

-not quite Maurice Chevalier

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# III

# Expect to See Chaos

## *Specific Military Systems and Technology*

Chaos Theory does not address every military system. However, while some authors still treat Chaos as a fashionable collection of new cocktail vocabulary, Chaos is neither a passing fad nor a mere metaphor. The extensive applications of Chaos to military systems make it imperative for today's decision makers to be familiar with the main results of the theory. This chapter is a quick review of the typical military technologies wherein one should *expect* to see chaotic dynamics. The chapter is broad by intent, since many more systems appear in chapter IV, where we start to apply Chaos results. The present discussion concludes with a necessary review of the theory's limitations as well as a summary of the implications of the pervasiveness of Chaos.

In the previous chapter we showed how little is needed to generate chaotic dynamics. If a system changes continuously in time—like the motion of vehicles and missiles—only three independent variables (three degrees of freedom) and some nonlinearity are required for chaotic dynamics to be possible. If a system changes in discrete jumps—daily aircraft sortie rates or annual budget requests—then any nonlinearity, as simple as the squared term in the logistic map, may provide a route to Chaos. These minimum requirements, present in countless military systems, do not *guarantee* chaotic dynamics, but they are necessary conditions.

Other common characteristics that make a system *prone* to Chaos include delayed feedback and the presence of external perturbations, or "kicks." An enormous number of military systems exhibit these features. One should expect Chaos in any system that includes feedback, fluids, or flight. The power of Chaos

theory lies in its discovery of *universal* dynamics in such systems. As this chapter proceeds from specific systems to general technologies, the reader should be alert for the similarities in diverse military systems.

*Naval Systems.* The Thompson and Stewart text on nonlinear dynamics includes a thorough discussion of the chaotic behavior of a specific offshore structure.[33] It reports a case history in which chaotic motions were identified in a simple model of a mooring tower affected by steady ocean waves. Mooring towers are being used increasingly for loading oil products to tankers from deep offshore installations. These buoys are essentially inverted pendulums, pinned to the seabed, and standing vertically in still water due to their own buoyancy. The concern in this "kicked" pendulum system is the potentially dangerous chaotic activity that occurs when a ship *strikes* the mooring. The number of impacts per cycle, which can be high, is an important factor to be considered in assessing possible damage to the vessel.

A 1992 Office of Naval Research report summarizes a series of studies identifying the sources of chaotic dynamics in other ocean structures: a taut, multi-point cable mooring system; a single-anchor-leg articulated tower; an offshore component installation system; and a free-standing offshore equipment system.[34] The author identifies key nonlinearities and analytically predicts transitions and stabilities of various structural responses. At the time of the report, experiments were still underway to verify the analysis. Ultimately, better ways to control these systems and to enhance current numerical models for these systems will be developed.

The naval applications of Chaos theory are not restricted, of course, to stationary structures. A recent graduate of the Naval Postgraduate School reports the use of nonlinear dynamics tools to control the motion of marine vehicles.[35] In this interesting application of Chaos results, the system itself does not display chaotic dynamics. However, the knowledge of common transitions away from stable behavior allows the author to improve the trajectory control of ships and underwater vehicles.

*Information Warfare.* As yet nebulously defined, the subdiscipline of military science known as Information Warfare certainly embraces a number of electronic systems subject to chaotic behavior. In many instances, chaotic dynamics contribute to the design of entirely new systems with capabilities made possible by Chaos theory. One large field of application is digital image compression. Simple equations that generate complicated distributions allow pictures to be expressed as compact sets of instructions for reproducing those pictures.[36] By transmitting the instructions instead of

all of the individual pixel values, thousands of times more information can be sent through the same transmission channels in a given period of time.

On large images and color images, these fractal compression techniques perform better than other current compression techniques.[37] In 1991, the decompression speed for the fractal method was already comparable to standard industry techniques. Even if this process does not become the new standard for real-time communication, it will probably drive the performance standards for other technology developments. Thus, this powerful technology is already making its way into military mapmaking and transmission as well as into real-time video links to the battlefield. Other potential applications will be discussed in the next chapter.

Two additional features of electronic Information Warfare make it ripe for Chaos applications. First, the high volume and speed of communication through computer networks include the best ingredients of a recipe for Chaos: modular processes undergoing endless iteration; frequent feedback in communications "handshaking"; and frequencies (on many scales) faster than the time it takes most systems to recover between "events" (messages, transmissions, and back-ups). Second, a likely place to anticipate Chaos is anywhere the digital computer environment approximates the smooth dynamics of real systems. Many iterated computations have been shown to exhibit Chaos even though the associated physical systems do not.[38]

*Assembly Lines.* A recent book on practical applications of Chaos theory presents a detailed explanation of where to expect and how to control chaotic dynamics in automatic production lines.[39] It focuses on a few subsystems: vibratory feeding, part-orienting devices, random insertion mechanisms, and stochastic (random) buffered flows. Possible military applications include robotic systems for aircraft stripping and painting and automated search algorithms for hostile missiles or ground forces.

Let us conclude this introduction to chaotic military systems by recalling the list of technologies in the 1991 Department of Defense Critical Technologies Plan.[40] This time, though, we can note the most likely places where these technologies overlap with the results of Chaos theory:

1. Semiconductor materials and microelectronic circuits—they contain all kinds of nonlinear interactions; semiconductor lasers provide power to numerous laser systems whose operation can destabilize easily with any optical feedback into the semiconductor "pump" laser.
2. Software engineering—refer to the discussion of Information Warfare, with feedback possible at unfathomable volumes and speeds.

3. High-performance computing—see items 1 and 2.
4. Machine intelligence and robotics—these require many varieties of control circuitry and feedback loops.
5. Simulation and modeling—chaotic dynamics are being recognized in numerical models that we have used for twenty years; look for more details in the next chapter.
6. Photonics—laser and optical circuitry may be subject to Chaos at quantum and classical levels of dynamics.
7. Sensitive radar—this often combines the instabilities of electronics, optics, and feedback.
8. Passive sensors—recall our night-light experiment.
9. Signal and image processing—fractals allow new advances in image compression.
10. Signature control—stealth technology, e.g., wake reduction in fluids.
11. Weapon system environment—this will be addressed in the next chapter's discussion of the nonlinear battlefield and "fire ant" warfare.
12. Data fusion—attractors and Lyapunov exponents can summarize new information for military decision makers.
13. Computational fluid dynamics—fluids tend to behave chaotically.
14. Air breathing propulsion—engines consume fluids and move through other fluids.
15. Pulsed power—power-switching requires circuitry with fast feedback.
16. Hypervelocity projectiles and propulsion—these will include guidance, control, and other feedback systems.
17. High energy density materials—they can undergo chaotic phase transitions during manufacture.
18. Composite materials—these pose the same manufacturing issues as item 17.
19. Superconductivity—superconductor arrays (Josephson junctions) are a classic source of Chaos.[41]
20. Biotechnology—living organisms are full of fluids and electricity, and Chaos.
21. Flexible manufacturing—this may include automated processes prone to Chaos.

## Limitations of Chaos Theory

It may seem difficult, after the previous section, to imagine a military system where we will *not* encounter Chaos. Let us, then, do a brief reality check to indicate some systems that *do not* seem to benefit from the results of Chaos theory.

In general, Chaos will not appear in slow systems, i.e., where events are infrequent or where a great deal of friction dissipates energy and damps out disturbances. For instance, we should not expect Chaos theory to help us drive a jeep or shoot a single artillery piece. (On the other hand, the theory may eventually guide our decisions about how to direct *convoys of Humvees* or how to space the timing or position of *many projectile firings*.) Similarly, Chaos theory offers no advice on how to fire a pistol, though it may pertain in the design of rapid-fire weapons.

Theoretical Chaos results are seriously constrained by the need for large amounts of preliminary data. To make any analysis of time series, for instance, we can make reasonable comments based on as few as one hundred data points; but the algorithms work best with a thousand or more.[42] Therefore, even if we are able to design reliable decisions tools for battlefield use, models that require hundreds of daily reports of enemy troop movements may be useless in a thirty-day war. While some hope remains for the prospects of increasing the speed and volume of *simulated* battlefield information, the mechanisms for using such simulations for real-time combat decisions remain to be developed.

One may encounter scenarios and systems with erratic behavior where a source of Chaos is not immediately evident. In this event, it may be necessary to examine different *scales* of behavior. For example, Chaos theory does not help study the flight of a single bird, free to choose where and when to fly. However, there *is* evidence of Chaos in how *groups* of birds flock and travel together.[43]

## Implications

The pervasiveness of chaotic dynamics in military systems forces us to be aware of sources of instability in system designs. We need to develop capacities to protect our own systems from unwanted fluctuations and to impose destabilizing dynamics on enemy systems. However, the next chapter will also present ways we can constructively exploit chaotic dynamics, to allow new flexibility in control processes, fluid mixing, and vibration reduction. We must remain alert for new perspectives on old data that were previously dismissed as noise. Perhaps more importantly, the universal results of Chaos theory open the door for new strategies—ideas we will discuss in the chapter ahead.

# IV

# How Can We Use the Results?

## *Exploiting Chaos Theory*

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One of the great surprises
to emerge from studies of nonlinear dynamics
has been the discovery that stable steady states
are the exception rather than the rule.

Siegfried Grossman and
Gottfried Mayer-Kress[44]

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At this point the reader should have some intuition for the common features of Chaos. An enormous number of systems exhibit chaotic dynamics; many of these systems are relevant to military decision making. But how can we use Chaos to make better decisions or design new strategies? Even if we accept the idea that Chaos *can* be applied to strategic thinking, shouldn't we leave this high-tech brainstorming to the analysts?

Absolutely not! As Gottfried Mayer-Kress points out, if we fail to learn the basic applications of Chaos theory, our naiveté could lead to unfortunate consequences. We may, for example, fall into the trap of thinking that successful short-term management allows total control of a system; we may have unnecessary difficulty in making a diagnosis from available short-term data; or we may

apply inappropriate control mechanisms that can produce the opposite of the desired effect.[45]

This chapter lays out practical results on how Chaos theory influences a wide range of military affairs. Sections of this chapter present specific suggestions on *how to apply* these results. Although the structure of each section may suggest that each concept or technique operates independently, like an isolated item in a tool kit, the application of Chaos theory unifies many of the previous results.

The chapter opens with a review of some Chaos results that are consistent with past thought and with good common sense. The meat of the chapter, of course, is a discussion of the new tools and options available to decision makers through the results of Chaos theory. Then, an introduction to fractals begins a section on applications that take particular advantage of the fractal geometries that appear in many chaotic systems. Finally, the chapter closes with a discussion of other issues, including the difficulties posed by making decisions about systems that include human input and interactions.

## Common Concerns

We should pause to consider the understandable concerns and objections of those who may be suspicious of "all this Chaos business." It is quite tempting to dismiss Chaos as an impractical metaphor, especially since many authors present *only* the metaphors of Chaos. Some toss around the Chaos vocabulary so casually that they leave no hope for practical applications of the results. Margaret Wheatley, for instance, offers Chaos only as a metaphor, hiding behind the argument that "there are no recipes or formulae, no checklists or advice that describe 'reality' [precisely]."[46] While it certainly is the case that no formula can track individual trajectories in a sensitive chaotic system, especially with human choice involved, *many* patterns are evident, many means of observation and control are available, and the trends of chaotic dynamics are sufficiently common that one can and should expect specific classes of behaviors and transitions in chaotic systems. Additionally, and unfortunately, many well-written Chaos texts target a highly technical readership; their useful results are not adequately deciphered for a larger community of potential users.

All the same, we already know that human activity is sensitive to small disturbances, that small decisions today can have drastic consequences next week, and that troops—like water drops—need rest between events. It is simply not obvious that there is anything *new* in the Chaos field. Why is it worth everybody's time just to learn a new vocabulary to describe the same old thing we have been doing for decades, or in some cases for centuries? Moreover, suppose we agree

that there is something new here. How can we *use* Chaos results? How can Chaos help us prioritize our budget or defeat an enemy?

Peter Tarpgaard offers a fine analogy that answers some of these concerns and offers a glimpse of the insight that Chaos theory brings to decision makers. Imagine what Galileo's contemporaries commented when they saw him depart for Pisa with a small ball and a large ball in his bag. "What's the use? You're going to climb the Leaning Tower, and drop the things, and they're going to fall. We know that already! You're not showing us anything *new*. Besides, even if it is new, how can we *use* it?"

Now consider the advance in knowledge when Newton derived precise expressions for the force of gravity. Among other things, Newton's laws of motion identified specific behaviors to expect when various objects are subjected to gravity's influence. By describing gravity's effects, Newton gave us the power to model them—if only approximately—and to assess their impact on various systems. In particular, we now know exactly how fast an object will fall, and we can figure out when it will land. With this knowledge, we can also predict and control certain systems.

Chaos theory brings comparable advances to decision makers. A number of researchers have developed techniques and tools that allow us to apply Chaos theory in physical and human systems; but these efforts are very recent, and a great deal of thought and study remains to be done. Enormous research questions are now open; several of these are mentioned in the following pages.

## Something Old, Something New

Various consequences of Chaos theory were recognized long before Lorenz uncovered the influence of nonlinearity in fluid dynamics. This lends some credibility to the results; as Clausewitz tells us, we need to compare new theories with past results to ensure their consistency and relevance. Many familiar topics in military thought disclose a relationship with Chaos theory. For example, the U.S. Army Manual FM 100–5 holds: "In the attack, initiative implies never allowing the enemy to recover from the initial shock of the attack."[47] This general strategy follows naturally from our observation of dripping faucets: Chaos results when the system is not allowed to relax between events. Similarly, Marine Corps doctrine specifically discusses the advantage of getting "inside" an opponent's "OODA" (Observe-Orient-Decide-Act) loops in order to decrease the appropriateness—and therefore the effectiveness—of the enemy's acts. The Marine Corps manual titled *Warfighting* (FMFM–1) involves many references to the consequences of sensitivity to current states and the unreliability of plans:

> We have already concluded that war is inherently disorderly, and we cannot expect to shape its terms with any sort of precision. We must not become slaves to a plan. Rather, we attempt to shape the general conditions of war; we try to achieve a certain measure of ordered disorder. Examples include:
>
> > . . . [channeling] enemy movement in a desired direction, blocking or delaying enemy reinforcements so that we can fight a piecemealed enemy rather than a concentrated one, shaping enemy expectations through deception so that we can exploit those expectations. . . .
>
> We should also try to shape events in such a way that allows us several options so that by the time the moment of encounter arrives we have not restricted ourselves to only one course of action.[48]

Likewise, as Michael Handel observed about the analysis of counterfactuals—alternative histories that might have occurred if key figures had made different choices—an important question is: how far can we carry an analysis of alternatives that were not actually pursued? He argued that the further ahead we consider, the less precision we should attempt to impose. In other words, the further we carry our counterfactual musings, the less reliable we render our analysis.[49] This is an expression of sensitivity to initial conditions, correctly applied to historical analysis.

We can see, then, that some of the consequences of Chaos theory do *not* present new findings for strategic thought. However, it is reassuring that these preliminary observations of Chaos theory are consistent with educated common sense and the conclusions of earlier researchers and thinkers. The mark of a good scientific hypothesis is that it adequately explains well understood phenomena and, additionally, it accounts for phenomena that was anomalous in (or unanticipated by) the hypothesis it is superseding.

## So What's New?

The applications presented in this chapter concentrate on methods, results, tools, and traits of dynamical systems that were not recognized, or even feasible, only thirty years ago.

The fact that deceptively simple-looking functions and interactions can produce rich, complicated dynamics constitutes a genuinely new insight. This insight grew in one case from the work of biologists' simple population models, like logistic maps, which were analyzed in greater detail by mathematicians. As a result, it was discovered that complex dynamics and outcomes do *not* have to come from complex systems. Apparent randomness and distributions of behavior

can be produced by very simple interactions and models. In another case, Edward Lorenz discovered that our difficulty in predicting weather (and many other chaotic systems) is not so much a matter of the resolution of the measurements as it is of the vulnerability of the system itself to small perturbations. In fact, global weather is so sensitive that even with a constellation of satellites measuring atmospheric data at one-kilometer increments across the entire globe, we could improve our long-range weather forecasts only from five days to fourteen days.[50]

So don't fire your meteorologists or your analysts! Simply to expect and recognize Chaos in so many real systems is progress enough. The best news is that many tools are available to understand and control chaotic systems. The tools of Chaos theory offer hope for discerning the key processes that drive erratic patterns such as the aircraft loss data shown in figure 2. J.P. Crutchfield highlights the importance of nonlinearity in developing those tools:

> [The] problem of nonlinear modeling is: Have we discovered something in our data or have we projected the new-found structure onto it? . . . The role of nonlinearity in all of this . . . is much more fundamental than simply providing an additional and more difficult exercise in building good models and formalizing what is seen. Rather it goes to the very heart of genuine discovery.[51]

A system's sensitivity often can be quantified and an estimate offered about how long predictions are valid. Only very recent advances in computers allow repeated measurements of such quantities as fractal dimensions, bifurcations, embeddings, phase spaces, and attractors. The results of these measurements are the information needed to apply the theoretical results. In this way, dynamical systems animate innumerable phenomena that have gone unmeasured until now; decision makers who are aware of the tools available to them can better discern the behavior of military systems.[52]

## *HOW TO APPLY*

While the results of Chaos theory improve our perspective of dynamics in military systems, the practical applications of Chaos go well beyond simple analogy. To highlight this point, the discussion of Chaos metaphors is postponed to the end of this chapter. The chapter focuses initially on specific processes, examples, and cases, with suggested insights and uses for the analytical tools presented earlier. Considering the applications of these results in one's own systems, it should be remembered that sometimes chaotic dynamics may be desirable, while at other times periodicity or stable steady states may be sought.

In other instances, one may want simply to influence the unpredictability in a system: increasing it in the adversary's system, decreasing it in one's own.

## Feedback

The results of Chaos theory help us to:

- know what transitions to expect when we add feedback to a system;
- suggest ways to adjust feedback;
- appreciate the wide range of dynamics generated by feedback in real systems.

There is nothing new about a call for awareness of feedback in physical and social systems. Many commentators, for instance, have remarked on the impact of real-time media reporting of combat events faster than DOD decision loops can operate. Similarly, one may consider the feedback imposed on an organization by requirements for meetings and reports. How often do these diagnostics "pulse" an organization? Yearly, monthly, weekly, daily? Do supervisors require periodic feedback, or do they allow it to filter up at will? Is the feedback in the organization scheduled, formatted, free-flowing, "open door," or a mixture of these? How intense is this occasional "perturbation"?

These are familiar issues for managers and commanders, but a grasp of chaotic dynamics prompts one to answer these questions with other equally important questions. What mixture of structured and free-form feedback works best in a particular system? What would happen if the frequency of meetings and reports were increased or decreased? What *transitions* in system performance should be expected? At what point, for instance, do too many meetings of an office staff generate instabilities in the organization? Or, in a crisis situation—theater warfare, rescue, natural disaster—what characteristics of the "system" make it more appropriate to assess the system every day, or every hour? This kind of idea was explored during a series of Naval War College war games. In these games, one out of every three messages was arbitrarily withheld from the commanders, without their knowledge. As a result, observers noted better overall performance in command and control processes.[53]

An awareness of the need for, and the sensitivity of, feedback in a system will make one more alert to the possible consequences of altering the feedback. Here, the biggest benefit of Chaos theory seems to be transitions that should be expected as various parameters of system feedback are adjusted. (Of course, this may or may not have validity in the real world.)

For example, if meetings or reports cause stress on an organization, several obvious parameters—frequency of feedback, length of reports, amount of detail or structure required in those reports, length of meetings, number of people involved in the meetings, and so forth—can be adjusted. Some experience with dynamical systems suggests that small changes or careful control of these parameters may suffice to stabilize some aspect of the system's performance. One new expectation we learn from chaotic systems is that small changes in control parameters can lead to disproportionate changes in behavior. Again, the idea of manipulating meeting schedules and reporting cycles is not new. However, the *expectations* for ranges of behavior and transitions between behaviors *are* new.

As a hypothetical illustration, suppose you observe changes in an adversary's behavior based on how often your surface vessels patrol near his territorial waters. Let us assume that your adversary bases no forces along the coast when you leave him alone, but he sets up temporary defenses when you make some show of force—say, an annual open–water "forward patrol" exercise. Assume, further, that when you double the frequency of your exercises to twice a year, you note a substantial change in your adversary's behavior. Maybe he establishes permanent coastal defenses or increases diplomatic and political pressures against you. You have cut the time difference between significant events (in this case, military exercise) in half and you observe a transition in the system. Now, it would be a silly idea to attempt to apply Feigenbaum's constant in this scenario and predict that the next transition in the adversary's behavior will come if you decrease the time interval by only 38 days. (Six months divided by Feigenbaum's constant, 4.67, equals 38.5 days.) On the other hand, the common features of chaotic systems suggest that—even though we have no model for the system—we should at least be alert that the *next transition* in this system could come if we increase the frequency of our exercises by only a small amount.

There may be few cases where one can afford the risk of testing such a hypothesis on a real adversary, though force-on-force dynamics like these could be simulated or gamed to reach significant, practical conclusions. We might consider, for instance, whether Saddam Hussein was playing a game just like this when he posted substantial forces along his border with Kuwait in 1994, while the United states military was busy with events in Haiti. Was he determining the increments of force size and timing that are necessary to provoke a U.S. military response? Perhaps Hussein was not applying Chaos theory to his strategic decisions, but we might analyze and game our own dynamics to see what increments of Iraqi force disposition would compel us to react. An understanding of chaotic dynamics ought to help us understand and control our response, selected from a

flexible range of options, because knowledge of Chaos helps us foresee the likely transitions when we change a system's control parameters.

Any one of the following questions would require a complete study in itself. However, they are presented to stimulate thought about the role of feedback, and transitions between behaviors.

The increasing availability of real-time information to decision makers amplifies concerns about information overload. How much detail does a leader require? How often? How much intelligence data does it take to saturate commanders and diminish their capacity for making effective decisions? What are the best ways to organize and channel a literal flood of information? The common transitions of chaotic systems suggest that it may be possible to learn how to control the flood by studying the effects of *incremental changes* in key parameters such as: volume of information, frequency of reports, number of sources involved in generating the data, and time allotted for decision making. Understanding the transitions from reasonable decision making to ineffective performance may help one tailor intelligence fusion systems for the benefit of commanders.

The relative timing of an incursion on an adversary's decision cycle may be more important than the magnitude of the interruption. Many successful strategies hinge on "getting inside the decision cycle" of the enemy. The idea, of course, is to take some action and then move with such agility as to make a subsequent move before an opponent has time to orient, observe, decide, and act in response to the first action. Chaos theory offers an important new insight into this basic strategy: we should expect ranges of different responses depending on how "tightly" we approach the duration of an OODA loop. That is, to outpace an enemy who operates on a twenty-four-hour decision cycle, revising the Air Tasking Order every eighteen hours may produce the same disorientation and disruption of the enemy as does revision on a twelve-hour or six-hour cycle. The planning timetable could then be selected on the basis of other objectives, such as speed, economy of force, efficiency, increased monitoring of combat effectiveness, or resupply requirements. The idea is that we should expect ranges of control parameter values where the system behavior is relatively consistent; but we also should note parameter ranges where small adjustments produce drastic changes in system response. This phenomenon is *not* sensitivity to initial conditions. Rather, it relates the sensitivity of the system structure and changes in parameters, or adjustments to the control knobs, if you will.

One final application to consider, in another area of the decision cycle: coordinating interactions with the news media during crises. It may be found that by adjusting the time intervals of wartime press conferences, for example, the effects of media feedback in our own decision loops may be mitigated without

having to resort to outright censorship. Periodic feedback, carefully timed, could contribute to desired behaviors in domestic systems, like channels of public support or an adversary's systems that tune in to American television for intelligence updates.

## Predictability

How does Chaos theory explain, illuminate, reduce, or increase predictability? Earlier sections of this paper refer to the unpredictable nature of chaotic systems: the irregular patterns in dripping faucets, rocking buoys, flickering lasers. Now we will consider the results that help us understand a chaotic system's erratic behavior.

While the paths of individual chaotic trajectories can never be accurately predicted for very long, knowledge of a system's attractors offers practical information about the long-term *trends* in system behavior. This section begins with a summary of powerful results that allow *prediction of the short-term behavior* of chaotic systems, even with no model. The section concludes with an explanation of the usefulness of attractors for assessing *long-term* system trends.

*Time Series Predictions.* We record—and sometimes analyze—large quantities of data at regular time intervals: daily closing levels of the Dow Jones Industrials, monthly inventory reports, annual defense expenditures. A list of measured data like this, along with some index of its time intervals, is called a *time series*. It may appear as a long printout of numbers, organized in a table or graph, indexed in time.

Now, if part of the list is missing, we might *interpolate* by various means to estimate the information we need. For instance, if we know a country's tank production was thirty vehicles three years ago, and thirty-two vehicles last year, we might guess that the production two years ago was about thirty tanks. To make this estimate we should first feel confident in the data we have on hand. We also should have some idea that industrial activity over the last few years was fairly constant. Further, there should be some reason to believe the production cycle is annual and not biennial. Finally, we should, perhaps, have access to a model that approximates this nation's production habits.

More often than not, though, we are concerned with *forecasting* issues such as how many tanks will a country produce next year? For such questions we must *extrapolate* and make some future prediction based on previous behavior. This is a perilous activity for any analyst, because the assumptions on which any models

are made remain valid only within the time span of the original set of data. At any point in the future, all those assumptions may be useless.

Unfortunately, predictions of behaviors and probabilities are an essential activity for any military decision maker; we have to muddle through decisions on budgets, policies, strategies, and operations with the best available information. Notably, however, the results of Chaos theory provide a powerful new means to predict the short-term behavior of erratic time series that we would otherwise dismiss as completely random behavior. Very briefly, here is the basic idea. If there were a time series with an obvious pattern, 2 5 7 2 5 7 2 5 7 . . ., the next entry in the list could be predicted with some confidence. On the other hand, if the time series displayed erratic fluctuations, as in figure 15, how could it be known whether there were discernible patterns to project into the future? Through the embedding process, Chaos analysts can uncover patterns and subpatterns that are not apparent to the naked eye and use that information to project the near-term behavior of irregular dynamics. In figure 15, for instance, where the time series approaches periodic behavior for a few cycles, embedding methods identify the places in phase space where these dynamics are most likely. This

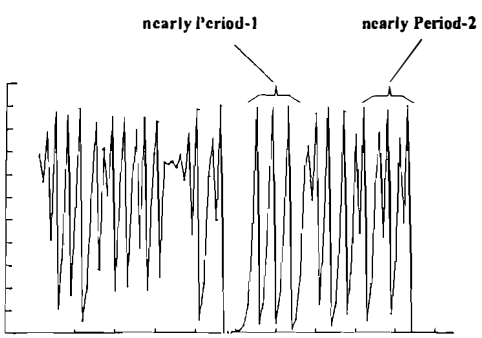


*Figure 15. Chaotic Time Series for the Logistic Map . . . . What Comes Next?*

technique has been applied to several complex fluids and thermal systems with tremendous success.[54]

The embedding technique, of course, does not work for all time series, and the predictions may hold for only a few cycles past the given data set. However, modern decision makers need to be aware of this tool for two reasons. First, without any help from Chaos theory, a wise person would not dream of trying to predict a single step of the wild dynamics illustrated in figure 15. The theoretical

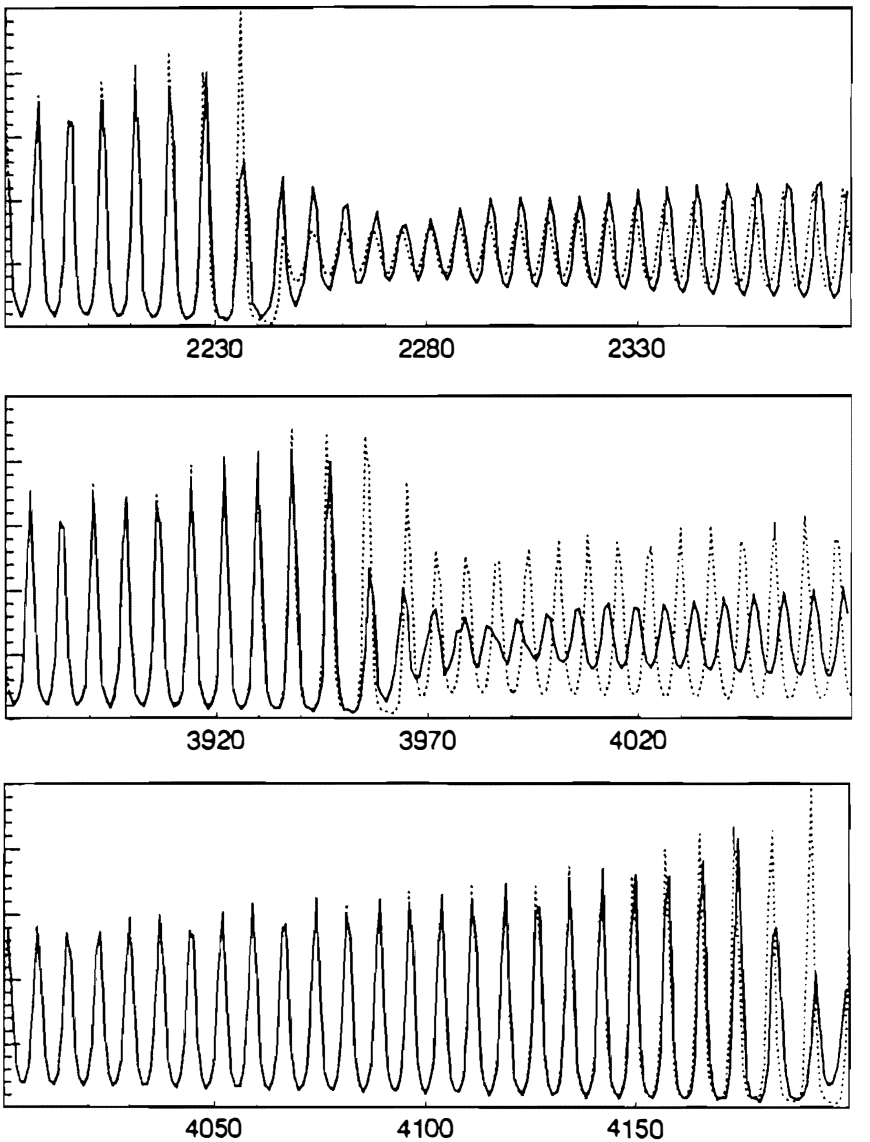


*Figure 16. Solid Lines Indicate Predicted Values. Dotted Lines Trace Actual Data.*[55]

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results give hope that one could make *reasonable projections* in systems previously dismissed as being beyond analysis. However, figure 16 includes samples of the kind of predictions possible with embedding methods. Given a thousand data points from which to "learn" the system's dynamics, the algorithm used here was able to predict fairly erratic fluctuations for as many as two hundred additional time steps.

In addition, embedding methods include *estimates of the error* induced by extrapolating the data, giving the decision maker an idea of *how long* the projections may be useful. (For detailed presentations of this technique, see, for instance, the notes from a 1992 summer workshop at the Santa Fe Institute.[56] Additional explanations also appear in a recent article by M. Casdagli, "Nonlinear Forecasting, Chaos and Statistics."[57] Both references outline the algorithms for near-term and global statistical predictions of chaotic time series.) Still other researchers have successfully applied similar methods to enhance short-term predictions by separating background noise from chaotic signals; this list includes Ott, Sauer, and Yorke,[58] J.D. Farmer,[59] and William Taylor.[60]

*Attractors and Trends.* It cannot be overemphasized that the sensitive character of chaotic dynamics denies any hope of predicting the long-term behavior of a system, regardless of how accurately its current state can be measured. On the other hand, any knowledge of a system's attractors gives considerable useful information to predict long-term *trends* in the system. For example, based on a glance outside we can probably tell whether we will need an umbrella to cross the street. We may even have enough information to make reasonable short-term decisions—like if we should go to the park this afternoon—even though the long-term weather remains unpredictable. On a larger scale, we can tell the difference in how to pack for a vacation in Hawaii versus a trip to Moscow, without any current weather information at all.[61] This is why it is fortunate that the weather behaves chaotically and not randomly. Otherwise, there could be no hope of making even short-term forecasts.

These simple examples illustrate how decisions can be based on some knowledge of system *trends*. The attractors of a dynamical system provide precisely that information. Whether an attractor is constructed from measured data or from extensive simulations, a system's attractor can illustrate trends that are not as intuitive as the simple weather examples above. Moreover, a well-drawn picture of an attractor vividly displays the relative amount of time the system spends in certain regions of its phase space.

Now, the kind of information discussed up to this point was available even before the advent of Chaos theory. However, the theory brings us several new

results when we are confident an erratic system is truly chaotic. First of all, simply by recognizing an attractor we regain some hope that we can understand and manipulate our system. After all, the attractor gives form and structure to behavior we otherwise would dismiss as random. Thompson and Stewart advise:

> Analysts and experimentalists should be vitally aware that such apparently random non-periodic outputs may be the correct answer, and should not be attributed to bad technique and assigned to the wastepaper basket, as has undoubtedly happened in the past. They should familiarize themselves with the techniques presented here for positively identifying a genuine chaotic attractor.[62]

Many practical pieces of information can be derived from our knowledge of a system's attractor. First, the relative amount of time the system spends on various portions of the attractor constitutes a *probability distribution*; an attractor could provide key probability information to a military decision maker in many scenarios. Secondly, if we find an attractor for a system, then any disturbances to the system's current state will still render its particular evolution unpredictable (envision a tire-swing or a vibrating space station). However, any transient behavior must die out, and the global trends of system behavior must be unchanged. That is exactly what the attractor describes: regions of phase space that attract system dynamics. Third, we have some hope of being able to predict or recognize the basins of attraction in a given system.[63] If we can prepare a battlefield or a negotiation scenario to our liking, we have some hope we can set up its initial state so the system proceeds under its own dynamics toward the trends of the attractor we desire.

Visualization of attractors also makes system transitions more apparent as we change control parameters. Recall, for instance, the return maps sketched for the dripping faucet (figure 6). It is important to notice that when the period-2 behavior first occurs, the pair of points in the attractor "break off" from where the single point used to be. A bifurcation occurs here; we find that the periods of these initial period-2 cycles are very close to the previous period-1 intervals. Thus, by tracking the attractors for various parameter settings, we not only observe the individual dynamics, but also discern additional information about the transitions between those behaviors.

Unfortunately, most real dynamical systems are not simple enough to collapse onto a single attractor in phase space. How can we understand and exploit *multiple* attractors in a single system? Here's an analogy: when my '85 Chevette starts up in the morning, it warms up at a relatively fast idle speed. This is one periodic (non-chaotic) attractor for the operation of my car engine with some fixed set of parameters. A few minutes later, when I tap the accelerator to release the choke,

the engine idles, but much more slowly. The system output has fallen onto a second periodic attractor. The system is the same, but an external perturbation "bumped" the system to a new, bounded, collection of states.

One may now wonder, is there any chance of exploiting the existence and *proximity* of two attractors in a system? Assume that the system of interest is the disposition of an enemy force, and suppose the current set of control parameters allows that system to evolve along either of two attractors, one of which is more to our advantage. Is it possible, by adjusting the control parameters available to us, to *manipulate the transitions* between these attractors, joining them, breaking them, building or destroying links between them? These questions may at first glance appear too metaphorical; but as one's facility with models and intelligence data increases, sometimes one finds that the answers to these questions bring extremely practical strategies to the table.

Chaos theory offers practical guidance for system predictability. Techniques like *embedding* make short-term prediction possible in chaotic systems. Also, these techniques *quantify the short-term reliability* of a given forecast. *Attractors* describe the *long-term recurrent behavior* of a system. The relative time spent in various states on the attractor defines useful *probabilities*. Images of attractors give indicators of the *features of system transitions*. And, finally, the presence of *multiple attractors* indicates the possibility of certain kinds of strategic options, although usually not their precise form.

## CONTROL OF CHAOS

One of the most powerful consequences of Chaos theory is that a chaotic system—whose behavior previously had been dismissed as random—can be influenced so that it becomes stable. Moreover, this is often possible without the aid of any underlying model. This capability has no counterpart in non-chaotic systems. Researchers have successfully controlled chaotic behavior in a surprising number of physical systems.

Three basic approaches have been demonstrated for Chaos control: regular periodic disturbances, proportional inputs based on real-time feedback, and trajectory "steering" based on models or approximations of the dynamics on an attractor.

The first control technique was demonstrated earlier: periodic output was induced in the chaotic dripping faucet by tapping a rhythm on the spout. In some respects, this technique is consistent with standard results of resonance theory that describe how external vibrations can excite certain natural frequencies in the system. However, in a chaotic system, *infinitely many* different periodic behaviors, not just combinations of the natural modes of system, are *guaranteed* to be possible.

The second control method, on the other hand, requires real-time measurements of the system's output in order to determine how far to adjust the selected control parameter. This is a generalization of the way you balance a long stick on the palm of your hand: you move your hand just enough, based on how you feel the stick leaning, and you manage to keep the stick upright. This method has the disadvantage of requiring a reliable feedback-driven control loop. The obvious advantage, though, is that stable output is achieved intentionally, not in the hit-or-miss fashion that sometimes characterizes control experiments of the first type.

The third control method was recently developed at the Massachusetts Institute of Technology (MIT). It requires extensive calculations in order to develop approximations to the dynamics on a system's attractor. Based on these approximations, the system parameters can be adjusted to guide a trajectory toward preferred regions of phase space. It has not been reported in any further experiments yet, but it is included to provide a peek at recent results.

These three techniques are the most practical means available to control systems that would otherwise exhibit Chaos; the methods allow imposition of different types of stability, depending on the application. For example, the stability generated may be a stable steady state (like balancing the stick), or it may be a stable periodic state (often desirable in laser systems). One also may entirely eliminate the possibility of Chaos by modifying the system in some way (see the discussion below on *process*). The key observation in all three techniques is that a chaotic attractor typically has kneaded into it an infinite number of unstable periodic orbits. Chaos control, then, comes from locking on to one of the infinitely many unstable periodic trajectories densely woven on an attractor.

Chaos control techniques offer many benefits. A chaotic system can be converted into one of many possible attracting periodic motions by making only small perturbations of an available system parameter. Better still, one method uses information from previous system dynamics, so it can be applied to experimental (real-world) situations in which no model is available for the system. Thus, control becomes possible where otherwise large and costly alterations to the system may be unacceptable or impossible.[64]

Several references describe the analytical details needed to implement these control algorithms. Ott, Grebogi, and Yorke perfected the technique that uses real-time feedback; current publications refer to this method by the authors' initials, as the "OGY method."[65] Since their initial report, they (and many others) have applied the OGY method to numerous systems, from classic chaotic systems, like Lorenz's weather model and the logistic map, to physical systems such as thermal convection loops, cardiac rhythms, and lasers. For example, figure 17

shows the stable steady state imposed on the logistic map compared to its usual irregular dynamics.[66] The OGY team has also applied this method of Chaos control to reduce and filter noise that is present in measured data.[67]

The other control technique, which is computation-intensive, was developed by Elizabeth Bradley at MIT.[68] Like the OGY method, this approach actively exploits chaotic behavior to accomplish otherwise impossible control tasks. Bradley's method, though, is more like a numerical interpolation. She successfully demonstrated her method on the Lorenz equations. Though it is not yet

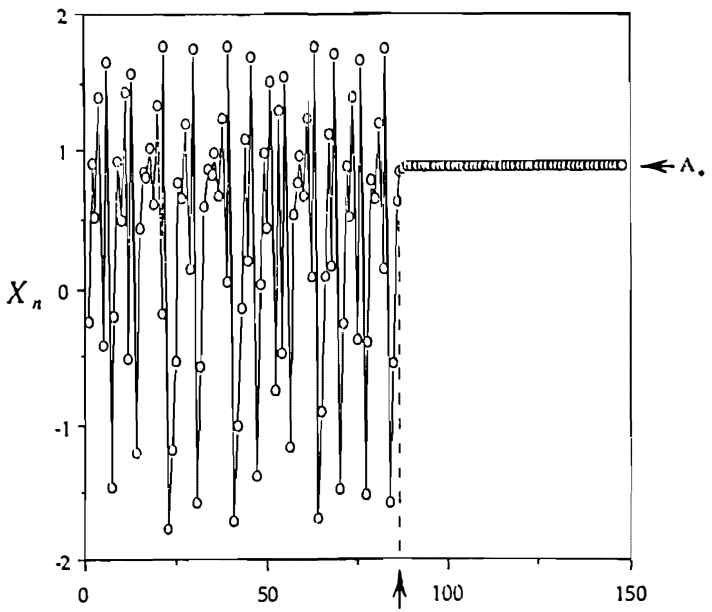


*Figure 17. The Logistic Map, Stabilized with Chaos Control.*[69]

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fully automated and requires a tremendous amount of data or a complete model, the technique shows great promise.

## Applications of Chaos Control

*Thin Metal Strip.* Early applications of the OGY method stabilized vibrations in a thin metal strip. Based on real-time measurements of the strip's position, the apparatus automatically adjusted the frequency and amplitude of input vibrations. This simple experiment confirmed the validity of Chaos control theory, stabilizing period–1 and period–2 behavior and switching between the two at will. These early successes highlighted the important consequences of Chaos control:

- no model was needed;
- minimal computations were required;
- parameter adjustments were quite small;
- different periodic behaviors were stabilized for the same system;
- control was possible even with feedback based on imprecise measurements.[70]

Most important, this method is clearly not restricted to idealized laboratory systems.

*Engine Vibrations.* Henry Abarbanel summarizes the results of several vibration control studies for beams, railroads, and automobiles.[71] He describes the use of automated software to discover the domains of regular and irregular motions in beams driven by external vibrations. This information is important to the study of lateral railcar vibrations, known as "hunting," which deform and destroy railroad beds. The hunting phenomenon—recognized for decades but never traced to its source—was shown to arise through the same period doubling transitions we saw in our dripping faucet and the logistic map! Understanding the source of these oscillations should lead to ways of mitigating the vibrations, saving significant costs in safety and maintenance.

In another case, S.W. Shaw's vibration absorber for rotating machinery successfully removed unwanted oscillations by prescribing paths for counterrotating dynamical elements. The induced motions precisely canceled vibrations in helicopter and automotive machinery. These nonlinear absorbers may appear soon in products of the Ford Motor Company, which sponsored the work.

*Helicopter Vibrations.* Chaos theory was applied recently, for the first time, to study flight test data from OH-6A higher harmonic control (HHC) test aircraft.

The HHC is an active system used to suppress helicopter vibrations. Most vibrations in the system are periodic, but evidence of Chaos was found. The presence of Chaos limits the ability to predict and control vibrations using conventional active control systems; but here, control techniques take advantage of the chaotic dynamics. Like the simple metal strip experiment, this approach uses only experimental data—no models. By extracting information from time series, one can find the limits of possible vibration reduction, determine the best control mode for the controlling system, and get vibrations under control using only a few minutes of flight data. These powerful analytical results reduced flight test requirements for the HHC; the same methods can be applied to other vibration control systems.[72]

*Mixing.* A South Korean company builds washing machines that reportedly exploit Chaos theory to produce irregular oscillations in the water, leading to cleaner, less tangled clothes.[73] Whether or not we believe this particular claim, we ought to consider military systems where effective mixing might be enhanced by Chaos control—for example, in the combustion of fuel vapors in various engines.

*Flickering Laser.* In a low-power laser at the Georgia Institute of Technology, Professor Raj Roy controlled the chaotic output of a laser by manipulating the laser's power source. Very slight but periodic modulations of the input power forced the laser into similar periodicity.[74] In this case, Chaos control was possible *without the use of feedback.* While the laser output was not driven to any specific target behavior, repeatable transitions were observed, from Chaos to periodicity, when Roy modulated a single control parameter.

Chaos control also finds a number of applications in circuits and signals.

*Ciphers.* In cryptography, as well as in many simulation applications, it often is necessary to produce large lists of pseudorandom numbers quickly and with specific statistical features. Chaotic dynamical systems appear to offer an interesting alternative to creating number lists like these, although sometimes more work is necessary.[75] Unfortunately, the same embedding techniques that allow us to make short-term predictions of chaotic behavior also make it easier to decode random-looking sequences. However, Chaos has other applications for secure communications.

*Synchronized Circuits.* Even the simplest circuits can exhibit sensitive, unpredictable long-term chaotic behavior. Yet with the correct amount of feedback, two

different circuits can be synchronized to output identical chaotic signals. This extraordinary result could prove useful for securing communications by synchronizing chaotic transmitters and receivers.[76]

*Taming Chaotic Circuits.* Elizabeth Bradley has completed software that takes a differential equation, a control parameter, and a target point in phase space, and approximates the system dynamics in order to drive a trajectory to a desired target point.[77] While computationally intensive, her approach has had good success controlling the Chaos in nonlinear electrical circuits. It takes information about dynamics on the attractor and translates that information into approximate dynamics that allow *control of individual trajectories.* As a result, this technique provides a more global approach to control processes.

*Human systems?* I have not yet seen Chaos control knowingly attempted on social systems, but consider, for instance, the options available for controlling the periodic dissemination of information to decision makers, both friendly and adversary. On the operational and tactical scales, we can envision many ways to apply periodic perturbations to a combat environment through action, inaction, deception, and information control. From a more strategic perspective, we can consider how regular negotiations and diplomatic overtures tend to stabilize international relations, while the absence of such measures allows relations to degenerate unpredictably. Depending on how such a system is defined, one might observe truly chaotic dynamics and new opportunities to control these dynamics. Of course, optimism must be tempered by emphasizing that active human participants can adapt unpredictably to their environments. However, a discussion follows shortly on the evidence of Chaos in human systems, offering some hope for applications.

The central idea is this: if a system is known to be (potentially) chaotic, then its attractor must contain an infinite number of unstable periodic trajectories. The presence of all these densely packed periodicities makes Chaos control possible.

There are further implications for system design, since it is possible not only to modify a chaotic system very efficiently with small control inputs but also to choose from a *range* of desired stable behaviors. Therefore, novel system designs are possible: we may be able to design a single system to perform in several dissimilar modes—like a guided weapon with several selectable detonation schemes, or a communications node with diverse options for information flow control. Current designs of systems like these usually require parallel components or entire duplicate systems in order to have this kind of flexibility. However,

knowing that Chaos is controllable, we now can consider new system designs with *Chaos built in*, so that various stable behaviors can be elicited from the exact same system through small, efficient perturbations of a few control parameters.[78]

## Chaos and Models

Why bother with applying Chaos to modeling? Some concerns are common to any debate about the utility of modeling. For instance, to increase doctrine's emphasis on the human aspects of war, Air Force Manual (AFM) 1-1 argues in detail that war must not be treated like an engineering project.[79] Also, there will always be trade-offs between the detail one would like in a model and the detail really needed. Gleick summarizes nicely: "Only the most naive scientist believes that the perfect model is the one that perfectly represents reality. Such a model would have the same drawbacks as a map as large and detailed as the city it represents, a map depicting every park, every street, every building, every tree, every pothole, every inhabitant, and every map. . . . Mapmakers highlight [only] such features as their clients choose."[80] And sometimes, even when good models are available, initial states can not be known (regardless of desired precision). For example, what initial conditions should be assumed for a complex model of the atmosphere, or an oil rig at sea in a developing storm? How can we hope to explore the responses from all possible starts?[81]

Sensitivity to initial conditions (SIC), of course, brings into question whether there is any utility at all in trying to run a computer model of a chaotic system. Why bother, if we know that *any* initial condition we start with must be an approximation of reality, and that SIC will render that error exponentially influential on our results as we move forward in time? Wheatley, among others, maintains a grim outlook on the whole modeling business in the face of SIC.[82] Yorke, however, has *proven* that even though a numerical chaotic trajectory will never be exactly the trajectory we want, it will be arbitrarily close to some real trajectory actually exhibited by the model itself.[83]

There are other reasons why we should struggle to understand the role of Chaos in modeling and simulation. The calculation of a time series' fractal dimensions is a means of assessing the number of effective independent variables determining the long-term behavior of a motion.[84] Simple computer models can be used to study general trends and counterintuitive consequences of decisions that otherwise appear to be good solutions. The results of even simple models will broaden our perspective of what *can* occur, as much as what is *likely* to occur.[85] Finally, Chaos results can help validate the behavior of models whose output appears

erratic. When we cannot match an individual time series, we can often match the distribution of behavior on an entire attractor.

*Chaos in the Simplest Models.* Even a brief survey of recent military models will reveal the importance of expecting Chaos in models and simulation. Ralph Abraham, for instance, gives a detailed analysis of what happens in his model of public opinion formation; his numerical exploration is a good demonstration of the process of wringing out a model. Chaos appears as he models the interaction of two hostile nations responding to the relative political influence of various social subgroups.[86] Other researchers at Oak Ridge National Laboratory have demonstrated a range of dynamical behavior, including Chaos, in a unique, competitive combat model derived from differential equations.[87]

Recent RAND research has uncovered certain classes of combat models that behave much like chaotic pendulums, and chaotic behavior appeared in the outcomes of a very simple computerized combat model. Preliminary studies offer ideas to better understand non-intuitive results and to improve the behavior of combat models.[88] For example, war game scenarios often produce situations where an improvement in the capability of one side leads to a less-favorable result for that side. Results like these have often been dismissed as coding errors. The correct insight, of course, is that non-monotonic behavior is caused by nonlinear interactions in the model. In the simple RAND model, reinforcement decisions were based on the state of the battle, and the resulting nonlinearities led to chaotic behavior in the system's output. The RAND team drew some interesting conclusions from their simulations:

- While models may not be predictive of outcomes, they are useful for understanding *changes* of outcomes based on incremental adjustments to control parameters.
- *Scripting* the addition of battlefield reinforcements (i.e., basing their input on time only, not on the state of the battle) eliminated chaotic behavior. This may not be a realistic combat option, but it is valuable information regarding the battle's dynamics.
- It is sometimes possible to identify the input parameters figuring most importantly in the behavior of the non-monotonicities (in this case, they were the size of the reinforcement blocks and the total number of reinforcements available to each side).
- *Lyapunov exponents* are useful for evaluating a model's sensitivity to perturbation.

In general, the RAND report concludes, "for an important class of realistic combat phenomena—decisions based on the state of the battle—we have shown that

modeling this behavior can introduce nonlinearities that lead to chaotic behavior in the dynamics of computerized combat models."[89]

John Dockery and A.E.R. Woodcock, in their detailed book, *The Military Landscape*, provide an exceptionally thorough analysis of several models and their consequences, viewed through the lenses of catastrophe theory and Chaos. New perspectives of combat dynamics and international competition arise through extensive discussions of strategy, posturing, and negotiation scenarios. They uncover chaotic dynamics in classic Lanchester equations for battlefield combat with reinforcements. They also demonstrate the use of many Chaos tools, such as Lyapunov exponents, fractals, and embedding.[90]

Dockery and Woodcock appeal to early models of population dynamics—predator-prey models—to model interactions between military and insurgent forces. The predator-prey problem is a classic demonstration of chaotic dynamics; the authors use common features of this model to simulate the recruitment, disaffection, and tactical control of insurgents. The analogy goes a long way and eventually leads to interesting strategic and tactical conclusions, illustrating conditions that tend to result in periodic oscillation of insurgent force sizes; effects of a limited pool of individuals available for recruitment; various conditions that lead to steady state, sustained stable oscillations, and chaotic fluctuations in force sizes; and the extreme sensitivity of simulated force strengths to small changes in the *rates* of recruitment, disaffection, and combat attrition.

In one of the many in-depth cases presented in *The Military Landscape*, patterns of dynamics in the simulation suggest candidate strategies to counter the strengths of insurgent forces. The model is admittedly crude and operates in isolation, since it can not account for the adaptability of human actors. However, the model does point to some non-intuitive strategies worth considering. For example, cyclic oscillations in the relative strengths of national and insurgent forces can result in recurring periods where the government forces are weak while the insurgents are at their peak strength. If the government finds itself at this relative disadvantage, and adds too many additional resources to strengthen its own forces, the model indicates that the cyclic behavior tends to become unstable (due to added opportunities for disaffected troops to join the insurgent camps) and paradoxically weakens the government's position. Instead, the chaotic model's behavior suggests carrying out moderately low levels of military or security activity to contain the insurgents at their peak strength, and await the weak point in their cycle before attempting all-out attacks to destroy the insurgent forces completely.[91]

## Process

Since many approaches to Chaos theory remain uncharted, we often find in reports of experiments and analyses that the *processes* followed are as instructional as the results. The laser system I studied at Georgia Tech with Professor Raj Roy is a good example.[92] We started with a low-power laser with output intensity that fluctuated irregularly when we inserted a particular optical crystal into the cavity. The crystal converts a portion of the available infrared light into a visible green beam, which is useful for many practical applications. Even though a previous set of equations described some of the laser's operation, no one had yet discovered the source of the fluctuations. Alternating between output from numerical models and the real laser, we modified the model, using reasonable basic physics, until the numerical results displayed Chaos. As a result, we identified the specific source of Chaos, and we were able to eliminate the chaotic fluctuations. This is one approach to consider for analyzing a system when a system exhibits Chaos but its model does not.

If, on the other hand, a model behaves chaotically but the real system does not, there are a few options. There may be, of course, fundamental mistakes in the model. A more subtle possibility is that one of the parameter values needs to be reduced (i.e., decrease the "energy" in the model) until the model matches reality. A third option, given confidence in the model, is to be alert for conditions when the real system might have different parameters. *Expect Chaos!*

If both the system and its model show Chaos, one should at least compare attractors, the distributions of the measurable output, like the histogram we drew in chapter II. Are the bounds on the attractors comparable? Do the densities of points on the attractors correspond? Once confidence in the model is developed, one may seek to draw explicit connections from *model parameters* to quantities that can be measured *in the system*. This is how to get control of the Chaos in a system.

These approaches have many potential applications, such as *generating distributions* for use in war-gaming models. If we can replace random algorithms in war-game models with simple chaotic equations that produce comparable distributions, we should find clues leading to the parameters that play the greatest role in the dynamics of given scenarios.

## Exploit Chaos for Strategies and Decisions

What is new about the application of Chaos results to strategic thinking? In general, our awareness of the new possibilities of *how* systems can behave brings

us definite advantages. Sometimes we will want Chaos. Perhaps an adversary's system will be easier to defeat if it is somehow destabilized. Cryptologists may prefer chaotic dynamics to secure their communications. On the other hand, many systems—signal transmissions, long-range laser sensors, and regular, predictable international relations—function better in stable, periodic conditions. Fortunately, Chaos theory also teaches us new ways to assure system stability through careful control of feedback.

Alan Saperstein pinpoints several ideas that Chaos theory brings to the strategic planner. First, many previous attempts to analyze international relations included notions of stability and instability that are *not* new in the Chaos results. However, previous models do not account for or produce extreme sensitivity to small changes in input or model parameters. Second, models have proven to be very useful in identifying trends, transitions, and parameter ranges where stability is prevalent. It follows that if incomplete models of international conflict show instability in given regions of parameter space, then more complete, "realistic" models are also likely to be unstable in larger regions of the parameter space, i.e., harder to stabilize. However, the converse is *not* true: if a given model representing a system is stable, then a more complex, more realistic model of the same system may still be unstable.[93]

The ideas in this section overlap somewhat with the previous sections on Chaos applications. The focus, though, is to assemble specific insights, options, and techniques available to military decisions makers and strategic planners. The examples proceed from specific results to general approaches. Among the many efforts to apply Chaos theory lie connections to military activities.

*Decision Making Tools.* Let us recapitulate some of the Chaos analysis tools available to military decision makers. These tools have surfaced throughout previous chapters in various examples and discussions:

- Given sufficient data, time series analysis allows short-term predictions, even in chaotic systems.
- Lyapunov exponents help to quantify the limits of predictions and measure a system's sensitivity to small disturbances. This information can help to prioritize various strategic options according to the relative unpredictability of their outcomes.
- Knowledge of common transitions in chaotic systems can suggest ideas for protecting and attacking military systems.
- Calculations of attractors depict distributions of outcomes, providing probability information to decision makers.

- Calculations of information dimension indicate the minimum number of variables needed to model a system. Moreover, a small value for dimension also represents strong evidence that the underlying dynamics are *not* random. A system with a non-integer dimension *must* contain nonlinearities (i.e., any previous models that are strictly linear must be incomplete).[94]

*Pattern Recognition.* In recent research at the Air Force Institute of Technology, the theory of embedded time series allowed James Stright to automate the process of identifying military vehicles from a few measurements of vehicle position and velocity. He also determined how long a data sequence is needed in order to classify accurately these moving objects. We can visualize the basic concept: the position of a drone aircraft with locked controls, for instance, should be far easier to predict than the position of a piloted aircraft conducting evasive maneuvers. So Stright generalized the idea of tracking objects as they move. At regular intervals, he noted a vehicle's position and velocity and logged that information in a vector. Evolution of these vectors constitutes an embedded time series; the patterns evident in this embedding allow characterization of typical vehicle behaviors. Stright verified his technique, correctly distinguishing the motions of five kinds of military vehicles.[95]

*Feedback Revisited.* Earlier, this paper discussed the role of feedback in chaotic military systems. Chaos theory brings new insights and options to strategies that include "pinging" an enemy system to see how it responds. Various parameters can be controlled to perturb an adversary's system—a large ground force, for instance. We can strike it periodically or unpredictably. We can change the magnitude (firepower), character (area versus directed fire), and frequency of our assaults. We can attempt to induce or reduce chaotic responses. We can reduce the amount of feedback in the system through operations security and information control. One might also envision particular attack strategies that apply our study of night-light dynamics to long-range perturbation of various enemy sensors.

Again, suppose we are forced to close a base or a port and replace our "forward presence" there with a "forward patrol" or "frequent exercise" or some periodic military presence. Chaos theory highlights relevant parameters that should be considered in strategic planning, such as the size of patrolling forces, the distances to the areas of interest, and the frequency of patrolling activities. Further, the dynamics common to chaotic systems warn of specific transitions to expect in an adversary's response as we vary any of those key parameters.

*Fire Ants.* Chaos applications in future strategies will follow in the wake of numerous revolutions in military technology. One such revolution may come in the form of "fire ant" warfare—combat of the small and numerous. It projects a battlefield covered with millions of sensors (the size of bottle caps), emitters (like pencils), microbots (like mobile computer chips), and micro-missiles (like soda bottles). These swarms will be deployed by a combination of pre-positioning, burial, air drops, artillery rounds, or missiles, and will saturate regions of the battlefield terrain.[96] Understanding the dynamics of weather systems and clouds suddenly becomes more than an academic exercise, because "fire ant" warfare produces a new combat climate: battlefields filled with new clouds that carry lethal capabilities. Anyone designing an enormous autonomous system like this, with millions of nonlinear interactions, had better be familiar with the complete range of possible dynamics as well as with the means to control and defeat such a system.

*SDI Policy.* Saperstein describes another use of Chaos in a numerical model to guide policy and strategy, carefully qualifying his findings in an intelligent numerical exploration and appropriately cautious use of modeling. The policy question was whether implementation of the Strategic Defense Initiative would tend to destabilize an arms race between the two superpowers. In this case, he relied on a nonlinear model to predict the outcomes of various options to help guide policy-making. Saperstein emphasizes that his model is a procurement model (not a force-on-force simulation) that includes inventories and production rates of various types of weapons. Among his conclusions were that a bigger qualitative change in the opponent's behavior comes with the introduction of defensive weapons, more so than with even drastic increases in annual ICBM production. Also, beyond his specific findings, his work exemplifies the delicate process of using models to guide decision making.[97]

*Operational Art.* Four fundamental questions face the commander of forces at the operational level of war. First, what military condition must be produced in the theater of operations to achieve the strategic goal? Second, what sequence of actions is most likely to produce that condition? Third, how should the resources of the force be applied to accomplish the desired sequence of actions? Fourth, what are the costs and risks of performing that sequence of actions?

The operational commander, of course, has access to the same tools available to any decision maker. Using these tools, the most direct applications of Chaos results are likely to be in answers to the second question, where Chaos tools can provide information about probabilities of outcomes. Notice, too, that when such informa-

tion is provided to a commander, it also represents feedback in his decision process, feedback that can produce transitions in his force's performance.

The second most likely use of Chaos will come in answers to question four, where the costs and benefits of various courses of action must be weighed. This paper proposes the use of Lyapunov exponents to help prioritize options based on the relative unpredictability of actions (see p. 30). No simulations or computer programs have yet been developed to implement this idea.

Moreover, Chaos theory may also address issues raised in question three, developing options for force application when one of the following conditions holds:

- We have access to enough *well-synthesized data* on an adversary's behavior to allow accurate near-term predictions of enemy actions;
- The opponent uses *sensors* or electronics that allow us to control enemy systems through feedback techniques;
- We face a *large force*, where we can exploit our knowledge of the distribution of behaviors in large interacting systems; or
- We engage in *prolonged combat*, with sufficient time for our observations of enemy behavior to reveal trends and patterns in enemy responses.

*Exploiting Chaos.* Overall, we need to anticipate chaotic dynamics so we can exploit them in our own systems as well as in enemy systems. A final caveat: besides the necessary reminder that combat participants can adapt in surprising ways, one should also remember that unpredictable changes in enemy dispositions can turn in the *enemy's* favor. In 1941, for instance, Japan managed to destabilize America's isolationist position by bombing Pearl Harbor. That this destabilization worked against Japanese hopes underscores the fact that the uncertainty produced by arbitrary disruption can lead to many unpredictable results, sometimes for better, sometimes for worse. Fortunately, the results of Chaos theory discussed above offer many strategic options beyond the mere arbitrary disruption of enemy systems.

## *Information Warfare Revisited*

Earlier we noted the vulnerability of communications systems to Chaos. Vast numbers of coupled electrical systems, many of which are controlled with feedback mechanisms, process immense quantities of information, all at the speed of light, with frequent iterations. Without the details of a given system, we cannot *guarantee* the onset of Chaos, but we definitely should expect chaotic dynamics in systems with those characteristics.

So far, we have identified the potential implications of enhanced data compression for Information Warfare, and the need to be aware of the numerical Chaos sometimes present in digital computations. I mention Information Warfare again in this section to tie together a few other applications discussed above. For one, Chaos applications in secure communications, in encryption, and in synchronized circuits will certainly play a part in Information Warfare. Also, Stright's automated algorithm for pattern recognition could eventually be applied to identify information "targets" just as it identifies physical targets.

## Fractals

Fractals have many more applications than merely serving as identifiers for time series with non-integer dimensions. Fractals play important roles in system scaling and in other image compression applications. First, we will examine some consequences of the multiple scales of dynamics present in real systems. Then we will see how researchers take advantage of these multiple scales to compress images with fractal transformations.

*Scaling.* We can gain new perspectives of military systems by considering dynamics on various physical scales, scales that become evident through the study of fractals. For instance, the reader can probably see Chaos right now in a system somewhere nearby: in the traffic patterns outside the building, in a stop sign wobbling in the wind, in the light flickering overhead, or on a computer display. However, and more certainly, there are many nearby chaotic dynamics occurring on physical scales that you probably don't care about, such as quantum fluctuations, or irregularities in the power output from a watch battery. The important idea is that we may sometime encounter system behavior we cannot explain because there may be key nonlinearities on a scale we have not yet considered.

Once we develop an awareness of the universality of many chaotic dynamics, we realize that some dynamics and physical properties occur on all scales in many systems, both natural and artificial. Gleick expresses this idea quite eloquently, guiding us to cases where we should expect to see scale-independent structures and dynamics:

> *How big is it? How long does it last?* These are the most basic questions a scientist can ask about a thing. . . . They suggest that size and duration, qualities that depend on scale, are qualities with meaning, qualities that can help describe an object or classify it. . . .
>
> The physics of earthquake behavior is mostly independent of scale. A large earthquake is just a scaled-up version of a small earthquake. That distinguishes earthquakes from animals, for example—a ten-inch animal must be structured quite

> differently from a one-inch animal, and a hundred-inch animal needs a different architecture still, if its bones are not to snap under the increased mass. Clouds, on the other hand, are scaling phenomena like earthquakes. Their characteristic irregularity—describable in terms of fractal dimension—changes not at all as they are observed on different scales. . . . Indeed, analysis of satellite pictures has shown an invariant fractal dimension in clouds observed from hundreds of miles away.[98]

Many other common systems exhibit the same dynamics on virtually any scale: hurricanes, fluid flow, airplane wings and ship propellers, wind tunnel experiments, storms, and blood vessels, to name only a few.

How does awareness of scaling properties broaden our perspective of military affairs? Just as we can conserve time and money by experimenting with scale models, we can sometimes resolve questions about a system's behavior by examining one of its components on a more accessible scale. For example, the electronic architectures of our war-game facilities nationwide are being configured to network as many sites as possible to conduct large-scale simulations. Unfortunately, the combat dynamics that are simulated at different facilities operate on different scales of combat: some are tactical simulations, some operational, and others strategic. War-game designers are currently faced with difficult questions concerning how to connect the flow of information among these participants on differing scales. The answer may eventually lie in a network based on fractal scaling of some kind.[99]

*Fractal Image Compression.* The need for data compression grows more apparent daily, as ships at sea saturate their available communication links, and users worldwide crowd a limited number of satellites and frequency bands.[100] Other requirements for information compression arise in large modeling problems, where physicists, for example, try to model cloud dynamics in simulations of laser propagation. One recent breakthrough in image compression came from Michael Barnsley's ingenious manipulation of fractals, leading to a process defined in his College Theorem.[101]

To compress an image of a leaf, for instance, Barnsley makes several smaller copies of the original image, and then he covers the original with the smaller copies. He tabulates all the *transformations* necessary to shrink, rotate, and translate those copies in order to cover the original leaf. That list of transformations is the only information necessary to reproduce the original image. Now, rather than transmit a picture of a leaf via pixel-by-pixel arrays of hue and brightness, we can transmit a brief set of instructions that allow the receiver to redraw the leaf very efficiently. By transmitting these short instruction sets, Barnsley's process compresses large color images by ratios in excess of 250:1. Not only has

Barnsley demonstrated this process with simple images, but he has proven that one can derive transformations for any image, up to the best resolution of a sensor.

The tremendous compression ratios by these fractal compression techniques make possible new applications in digitized maps for numerous uses, including devices for digitized battlefield equipment and avionics displays. Moreover, the end-product of this transmission process is, in fact, an attractor of a chaotic system, so it contains density information about how often a given pixel is illuminated by the receiver's redrawing program. Among other uses, this local density information translates into useful data for the physicist interested in propagating lasers through clouds.

Barnsley's company, Iterated Systems, Inc., has already won several Army and Navy research contracts to make further advances with this compression technique.[102] One of the resulting products was a patented algorithm for pattern recognition, with the potential to develop automated means to prioritize multiple targets for a weapon system. Iterated Systems has also used fractal compression to transmit live motion video across standard telephone lines, a capability with numerous operational applications.[103]

## Metaphor

> You don't see something
> until you have the right metaphor to let you
> perceive it.
>
> Robert Shaw[104]

This section deliberately is short. Chaos does offer powerful metaphors that lend genuinely new perspectives to military affairs, but since we have access to so many practical applications that flow from Chaos theory, I will minimize this brief digression. The main idea is that the metaphors of Chaos bring a fresh perspective—not just a new vocabulary for old ideas. This perspective comes with an awareness of new possibilities: new information (fractal dimensions, Lyapunov exponents), new actions (feedback options, Chaos control), and new expectations (stability, instability, transitions to Chaos).

In a recent attempt to use Chaos metaphors for new historical perspectives, Theodore Mueller of the Army War College depicted the *Mayaguez* crisis as the result of a system destabilized due to its sensitivity to small disturbances. He used

the image of an attractor to describe departures from the "range of expected behavior" for an adversary.[105] In another case, a Santa Fe Institute study generalized the results of classic predator-prey equations and drew interesting politico-military analogies from simple models. The study made a rough comparison of how the onset of epidemics, modeled in these equations, compares to social dynamics that may spark political revolutions.[106] More case studies applying Chaos metaphors are likely to follow, as the military community grows familiar with the theory's more practical results.

## The Human Element—Chance, Choice, and Chaos

*Problems.* Certainly, Chaos theory can boast an impressive record in mechanical and numerical applications, but can we, and should we, use these results in systems that include human input? How do we reconcile Chaos results with the apparently random dynamics of unpredictable human decisions, the transient nature of social systems, or the Clausewitzian interaction of adversaries in combat?

Some of these questions necessarily arise in any debate over the utility of modeling a system that includes human decisions or responses. We have cause for suspicion, in particular because any analysis of social systems assumes we are able to recognize and predict trends in human behavior. If such predictions are possible, where does that leave our perspective of choice and free will?

Even if we suspend our disbelief long enough to explore candidate models for human behavior, we face significant obstacles to executing our analysis. Aggregate data sufficient for strong empirical tests simply do not exist for many important social systems. Social systems are not easily isolated from their environment. These systems encompass huge scales in time and space, vast numbers of actors, cost variables, and ethical influences. The laws of human behavior are not as stable as the laws of physics.[107]

This section argues that Chaos theory does shed light on human behavior that is relevant to military affairs. Certainly, Chaos is only one of the many rich dynamics we can observe in human behavior. However, we will focus on some of the constraints on human behavior that give us reason to look for insight from chaotic modeling and simulation efforts. Next, we will present recent evidence of the presence of Chaos in human behavior. Finally, we will offer some preliminary ideas on how additional Chaos results can be applied to military affairs.

*Hope.* Let us look at some sources of hope for understanding human systems with the help of Chaos theory. First of all, despite our seemingly unlimited

capacity for creativity, we will always make decisions within constraints imposed by limited resources, limited time, personal habits, and external pressures such as policy and opinion. Some of our constraints stem from periodic cycles in our environment, both natural and fabricated: twenty-four-hour days, human physical endurance, seasonal changes, planetary motion, tides, revisit times for a satellite with a small footprint, equipment reliability and maintenance, replenishment and resupply, time cycles necessary to conduct battle damage assessment, budget cycles, and periodic elections. This list is not intended, of course, to promote astrological applications in strategic planning. However, we have seen numerous examples where periodic perturbations can drastically alter a physical system's dynamics, causing significant shifts toward or away from stable behavior. The pervasiveness of these constraints—often periodic constraints—gives us cause to expect chaotic dynamics even in systems influenced by human decisions and responses.

Another reason to be optimistic about Chaos applications in human behavior comes from the very nature of attractors: within an attractor's basin, transient behavior will die out and a system will be found only in states that lie on the attractor. Even if the system is perturbed at a later time, it must return to the attractor. Evidence exists that points to the occurrence of non-random chaotic dynamics in human systems. Those dynamics, in turn, imply the presence of attractors for those systems. This does *not* imply that there is no influence of choice and chance in these systems. Rather, in these cases, human decisions represent one of the following influences: *perturbations* of behavior which would otherwise remain on an attractor; *changes in the distribution* of behavior, i.e., tendencies of the system to stay on any particular portion of the attractor; or *choices among multiple attractors* that exist in a single system.

A personal guess is that we will eventually find phase spaces with *multiple attractors* to serve as the model for the various options available to us or to an adversary. As a playful analogy, think about the possible "state" of your mind as you read this essay; suppose we can somehow characterize that state by measuring your thoughts. Is there any hope of controlling or manipulating that system? If you think not, consider what happens to your thoughts when I tell you, "DON'T think of a pink elephant." Whatever attractor your mind was wandering on before, did your thoughts pass through my "pink elephant" attractor, even momentarily? I contend that we have hope of modeling, understanding, and perhaps controlling some features of human influences in military affairs, perhaps only briefly, but long enough to enhance the planning and execution of numerous military activities from acquisition to combat.

In a study of two species of ants, whose social dynamics are much easier to observe than human ones in a controlled environment, Nobel Prize winners Gregoire Nicolis and Ilya Prigogine give us some additional hope for making analyses of human systems.

> What is most striking in many insect societies is the existence of two scales: one at the level of the individual, characterized by a pronounced probabilistic behavior, and another at the level of the society as a whole, where, despite the inefficiency and unpredictability of the individuals, coherent patterns characteristic of the species develop at the scale of an entire colony.[108]

While they draw no premature conclusions about the immediate consequences of these results for human behavior, Nicolis and Prigogine offer this evidence as reason to be optimistic about the possibility of analyzing and controlling group dynamics. Ralph Abraham also reminds us that we can study human decisions through game theory, where chaotic dynamics have surfaced in the conduct of different games. A number of complex models are already making significant progress in explaining the actions of, and reactions among, multiple players.[109]

*Evidence of Chaos.* Is there evidence of chaotic behavior in human systems? The sort of symptoms one should be looking for are: a well-defined system, a clear list of observables to measure, aperiodic changes in those observables, bounded output, sensitivity to small disturbances, evidence or knowledge of nonlinear forces or interactions, attractors with fractal dimension, and small, non-integer information dimension. Several research papers report findings of many of these symptoms in historical data as well as in simulations using models that correspond well with observed human behavior.

Robert Axelrod, for example, has created a model that predicts how elements in a system group themselves into patterns of compatible and incompatible elements. He modeled nonlinear interactions with basins of attraction that predict how multiple actors in a scenario form opposing alliances. Typical aggregation problems where his results may apply include international alignments and treaties, alliances of business firms, coalitions of political parties in parliaments, social networks, and social cleavages in democracies and organizational structures. The basic inputs to his model are a set of actors, the size of each nation-actor, their propensity to cooperate with each other, partitions (physical and otherwise), the distance between each pair, and a measure of "frustration" (how well a given configuration satisfies the propensities of a country to be near or far from each other actor). Axelrod's theory correctly predicts the alignment of nations prior to World War II, with the exception that Poland and Portugal

were mistakenly placed on the German side. He also had comparable success predicting how computer businesses would align behind various market standards, such as the selection of operating systems. His prediction correctly accounted for 97 percent of the total number of firms in the sample.[110]

In another discovery of Chaos in social systems, Diana Richards presented several examples of experimental and empirical evidence in strategic decision making. In one example, she expanded a simulated prisoner's dilemma game to illustrate possible dynamics in collective decision making in politics and economics. In this model, nonlinear interactions arose because the players' decisions depended on their responses to actions in previous steps. She allowed each of two simulated participants to choose from a hundred options; various stable and chaotic dynamics resulted when she iterated the model.

On one hand, Richards emphasizes the difficulties in verifying such a model because of the problem of collecting real data over as many repetitions as she can easily simulate numerically. On the other hand, she was able to apply time series analysis to uncover chaotic dynamics in historical data. In particular, she discovered evidence of Chaos in U.S. defense spending (as a percentage of total federal spending) between 1885–1985, and in the number of written communications per day (between and within governments) during and following the Cuban missile crisis, October 1962 to January 1963.[111] Again, the presence of Chaos in these systems does *not* indicate that their behavior is completely predictable; but the number of variables which drive their dynamics may be much smaller than our intuition might suggest, and we may have a better chance of modeling, understanding, and controlling these situations than previously thought possible.

A significant study of historical military data was completed by a team of students at the Air Command and Staff College (ACSC) in 1994. Their report appears to be the most thorough research to date that examines historical data with the tools of Chaos theory. Their calculations of fractal dimensions and return maps present conclusive evidence of Chaos in tactical, operational, and strategic dynamics of military activity, as shown in aircraft loss data for the entire Vietnam War (see figure 2), Allied casualty data during their advance through western Europe in World War II, and historical U.S. defense spending (with results consistent with the Richards report mentioned above).[112]

Recent investigations of well-known models in system dynamics have revealed previously unsuspected regimes of deterministic Chaos. One outstanding example is John Sterman's comparison of two numerical models to controlled tests with human players. The first scenario is a production-distribution model of the Beer Distribution Game, where subjects are asked to manage a product inventory in the face of losses, delays in acquiring new units, multiple feedbacks, and other

environmental disturbances. Despite the difficulties of conducting controlled experiments, Sterman found that the human subjects' behavior is described fairly well by the model dynamics. This direct experimental evidence that Chaos can be produced by the decision-making behavior of real people has important implications for the formulation, analysis, and testing of models of human behavior.[113]

Sterman's second scenario simulates a long economic wave in which players adjust inventory orders in response to long-term indicators of supply and demand. The simulated business begins in equilibrium; an optimal response to the provided indicators actually returns the system to equilibrium within six annual cycles. However, of the forty-nine subjects tested, none discovered the optimal behavior, and the vast majority of subjects produced significant oscillations, many of which showed evidence of Chaos.[114]

Further practical evidence of Chaos in individual behavior is discussed in recent NASA-sponsored research. In lab tests, researchers took electroencephalogram (EEG) measurements of a human in efforts to characterize the "error prone state" of, say, a tired pilot. Are some individuals more prone to enter these states than others? What is the EEG signature of such a "hazardous state of awareness"? They found that standard statistical tools could not distinguish the EEG signal of an individual engaged in various activities from mental arithmetic to image identification. However, the average point-wise (fractal) dimension of the EEG did distinguish the different types of activity. This work has the potential to develop automated monitoring of pilots in flight to warn them of decreased alertness. More generally, this gives hope of applying Chaos results in order to understand the dynamics of human behavior.[115]

*Implications.* There are still very few documented attempts to apply Chaos results to social systems, due partly to the novelty of Chaos theory, and partly to the practical problems discussed above. However, many authors have noted important implications of the evidence of Chaos in social systems. Hal Gregersen and Lee Sailer, for instance, draw two principal conclusions. First, social studies rely too much on single measurements of population cross-section; we need to focus instead on data taken incrementally over long periods of time. Second, in addition to standard statistical analysis, we need to recognize Chaos and use the new tools of dynamical systems.[116]

The ACSC research team also offered a good summary of the implications of chaotic dynamics in the data they studied:

- Many erratic systems are at least partly deterministic, so do not throw out data that appears to be noisy.
- The presence of Chaos requires models that include nonlinear interactions.

- The inclusion of nonlinearity implies that models are likely to have no analytical solution, so do not throw out the computers (or the analysts)!
- Fractal dimensions estimate the minimum number of variables needed to build models.
- Some regions of phase space are more sensitive than others; Chaos tools can help identify those different regions.
- Tracking the patterns in attractors also helps identify excluded regions of behavior.[117]

*How to Apply the Results.* Ultimately, we will need to verify any theoretical claims by comparing them with real systems. In light of the problems of matching numerical models to human behavior, we are left with two basic options. We can construct and analyze formal models only, comparing model results to historical data; or we can develop lab experiments with human subjects interacting with computer-simulated social systems, or "microworlds."[118]

These two options still leave much room to apply Chaos theory to the study of social systems. For instance, Gottfried Mayer-Kress set up a simple model of a superpower arms race and discussed several immediate consequences of his simulated results. Surprisingly, the model gave little or no warning of the onset of political instability via the usual transitions to Chaos.[119] Thus, the use of a chaotic model can indicate uncommon transitions to unstable behaviors, providing new insight to what can happen in reality, despite the crudeness of the model.

How might we specifically adapt Chaos results to organizational behavior? A recent article discusses The Conference Model™, a series of conferences structured to help a large group implement effective reorganization.[120] The process entails several carefully structured steps, involving a large number of group members, that encourage "ownership" of the process—comparable to current DOD Total Quality policies and processes. The authors report significant success with their process; it can be couched in terms of Chaos theory to shed light on outcomes to expect from their suggestions for further research.

To begin, the researchers define their system well: basically, it is an organization with fixed membership, divided into subgroups of managers and employees, planners, and doers. The key parameters are the number of people of the various groups involved in the planning activities, the number of meetings, the number and timing of follow-up activities. The measures of effectiveness include the time required to design the organization's plan for change and the time taken to implement the changes.

One of the issues raised in this study is, what is the outside limit on the number of people who can attend a conference? This question could be recast as an issue about the ranges of possible dynamics as the key parameters are changed. For instance, what transitions are likely as the number of participants involved in the planning process gradually decreases from 100 percent of the organization? At what point do we note a substantial decrease in the effectiveness of the plan's implementation? The universal results of chaotic dynamics suggest we should expect specific transitions (e.g., oscillations of some type) sometime before we reach the point of total failure of the planing process.

John Sterman's conclusions about his lab experiments provide a good summary of both the tremendous potential and the unresolved issues of applying Chaos to human systems. Test results, he notes, show that participants' behavior can be modeled with a high degree of accuracy by time-tested decision rules. New chaotic dynamics have been observed, in well-accepted models, for reasonable parameter ranges. The evidence strengthens the arguments for the universality of these phenomena. However, the short time scales of important social phenomena often render the utility of Chaos questionable. The role of learning is difficult to gauge, e.g., in the experiments discussed here, thousands of cycles are simulated; however, evidence shows that subjects began learning after only a few cycles. Most important, the results demonstrate the feasibility of subjecting theories of human behavior to experimental test in spite of the practical difficulties.[121] Chaotic dynamics will continue to surface in future investigations of human systems. We need to be prepared to recognize those dynamics when they occur.

## Chaos and Military Art

This chapter compiles substantial evidence of predictable, controllable dynamics governing many aspects of military affairs. Does it say there is no room left for military art? Quite the contrary: while chaotic dynamics are sufficiently universal to revolutionize our profession, Chaos theory is only one of many necessary tools. Where is the individual art of the commander still evident? A good simulation, for instance, or a good summary of intelligence estimates may draw a clear picture of an adversary's attractor. Perhaps the image displays trends in force deployment, in aircraft ground tracks or in satellite footprints. However, an attractor only helps express probabilities; the commander still requires a sense of operational art to evaluate those probabilities in various courses of action, assess the risks of diverse options, and choose a single course of action.

## *What Do You Want Us To DO?*

This nontrivial question was posed by a concerned audience member after I presented an introduction to Chaos at ACSC. I am convinced we must *not* leave Chaos to the analysts and wait a few years for more results. I encourage you to gain confidence that you can learn the essential material from good readings and patient thought. You can discern good sources from bad, using the "Chaos con" tips and good sense. You can build better intuition for what to expect, what Chaos can do for you, when you need to consult your in-house analysts, when you need to pay a contractor to do more research, and when you should tell the contractors to go to the library and do their own homework on their own money. You should develop an expectation of, an anticipation for, chaotic dynamics in the motion and changes you observe daily.

Read confidently. When you write, use the vocabulary with care, and at least avoid the pitfalls outlined in my section on the Chaos con! However, *do* write. Publish your progress and successful problem-solving and models to show others your process for applying the results of Chaos theory. Above all, be aware of the avenues that are opening due to the far-reaching results of Chaos theory.

David Andersen outlines several additional points he feels should be highlighted when we teach anyone about chaotic dynamics. These points certainly offer good advice for any decision maker considering the application of Chaos to military affairs.[122] Andersen urges us to understand phase plots in order to develop an intuition for Chaos. We should learn to distinguish between transient and steady-state dynamics. We must be ready to spend time computing. He recommends that we take the time to get some theoretical background. Most significantly, we should learn to recognize when Chaos might be near and how to diagnose it when it does appear.

## Chapter Summary

Tremendous opportunities await us in the numerous realms of Chaos applications. We have access to insights and strategic options that were unimagined only twenty years ago: universal transitions in system behavior through the careful control of system feedback; new capabilities to predict short-term dynamics and long-term trends; options for controlling erratic systems previously dismissed as random; extraordinary advances in computations that enhance our communications capacity and improve our simulations. In the end, despite reasonable concerns about the utility of modeling, in general—and the analysis of human

systems, in particular—we find a wealth of new information, actions and expectations made possible due to the continuing advances in the understanding of Chaos theory.

# *Part Three*

## What Next?

### *A Road Map to More Chaos*

# V

## Suggestions for Further Reading

This chapter summarizes the best resources I encountered during my research. Many Chaos books have appeared in just the last four years; this review only scratches the surface of this pool of published resources, not to mention numerous videos and software. My aim is to offer some guidance to instructors on sources to recommend for additional reading, to students on the best leads for more detail, and to all readers curious about the individuals and organizations who are researching and writing in diverse areas.

The focus of this paper has been to build a bridge from Chaos theory to *your* areas of interest; the following books and periodicals offer interesting destinations for you to consider. The most thorough, well-developed readings came from Gottfried Mayer-Kress (numerous articles), Woodcock and Dockery (*The Military Landscape*), John D. Sterman (writing in a special issue of *System Dynamics Review*), James Gleick's classic, *Chaos*, and a special issue of *Naval Research Review* devoted to Chaos research sponsored by the Office of Naval Research. Further discussion of these and other references follows.

James Gleick, *Chaos: Making a New Science* (New York: Viking Penguin, 1987). Gleick composes vivid descriptions of the people and places at the roots of Chaos theory. He interlaces narratives with detailed personal interviews. This book is very readable, and it assumes no technical background. It is not the best place to learn the details of Chaos—the concepts presented are very general—but it is a pleasant exposition of the wonder of discovery, the universality of Chaos, and its range of applications. Take the time to read all the endnotes where Gleick hides additional interesting facts. A great piece of storytelling.

Heinz-Otto Peitgen, Hartmut Jürgens, and Dietmar Saupe, *Chaos and Fractals: New Frontiers of Science* (New York: Springer-Verlag, 1992).

The authors have compiled a veritable encyclopedia of Chaos. The text is very readable, assumes little technical background, and explains fascinating connections among diverse Chaos applications. If you put only one Chaos book on your shelf, this should be it.

*System Dynamics Review*, vol. 4, nos. 1–2, 1988.

This special issue assembles a fine collection of articles that discuss important issues of Chaos theory in great depth. The topics range from the very practical to the philosophical. John D. Sterman, for instance, opens the issue with a well-written introduction that surveys the basic concepts and results of Chaos theory; he also contributes a strong paper on "Deterministic Chaos in Models of Human Behavior: Methodological Issues and Experimental Results." This is another must-read resource.

J.M.T. Thompson and H.B. Stewart, *Nonlinear Dynamics and Chaos* (New York: John Wiley & Sons, 1986).

The authors aim this superb text at engineers and scientists, analysts and experimentalists. They require as background only "a little familiarity with simple differential equations." Step-by-step, they introduce Chaos, what to expect, and how to interpret data sets with irregular behavior; they use numerous helpful pictures and graphs. In addition, they present a healthy range of applications, focusing on the ways simple models can generate complicated dynamics in slender, vibrating structures; resonances of off-shore oil production facilities; large-scale atmospheric dynamics; particle accelerators; chemical kinetics; heartbeat and nerve impulses; and animal population dynamics. They also include a fantastic bibliography with more than four hundred entries. This is a great book from which to learn Chaos theory.

John T. Dockery and A.E.R. Woodcock, *The Military Landscape* (Cambridge, England: Woodhead Publishing, 1993).

This book presents an exceptionally detailed analysis of several models and the implications of their dynamics viewed through the lenses of catastrophe theory and Chaos. New perspectives of combat dynamics and international competition surface during the analysis of the models' behaviors. The authors discuss extensive applications in strategy, posturing, and negotiation. In one of their many simulations, they uncover chaotic dynamics in the classic Lanchester equations for force-on-force combat, with reinforcements. They demonstrate the use of

many Chaos tools, and they take great pains to show relationships among the tools. Overall, this book includes more analytical details than most recent reports, and it is a thorough review of many models that exhibit chaotic dynamics.

John Argyris, Gunter Faust, and Maria Haase, *An Exploration of Chaos*, Texts on Computational Mechanics, Vol. VII (New York: North-Holland, 1994).

Offered as an introductory text on Chaos theory, this book targets "aspiring physicists and engineers." A good deal of general theory precedes a review of physical and mechanical applications. The authors claim to assume no deep mathematical background, but the reader really needs more than a casual familiarity with differential equations and vector calculus. The book has several strengths: a detailed discussion of the logistic map; a nice compilation of classes of bifurcations; an interesting analysis of bone formation and regrowth. The applications are presented in fine detail, making the results reproducible for interested readers. Most importantly, the authors outline a general process of theoretical and numerical investigation appropriate for technical applications of Chaos results. They conclude with a spectacular bibliography of primary technical sources.

Richard A. Katz, ed., *The Chaos Paradigm: Developments and Applications in Engineering and Science*, American Institute of Physics (AIP) Conference Proceedings 296, Mystic, Conn. (New York: AIP Press, 1994).

This is a terrific survey of current research sponsored by the Office of Naval Research and the Naval Undersea Warfare Center. The list of participants is a useful "Who's Who" of many current research areas; the articles sample the diverse fields where DOD engages in active research. Anywhere from two to four brief articles cover each of the following topics: Mathematical Foundations of Chaos, Mechanical Sources of Chaos, Turbulence, Control of Chaos, Signal Modeling, Noise Reduction, Signal Processing, and Propagation Modeling.

Todor Tagarev, Michael Dolgov, David Nicholls, Randal C. Franklin, and Peter Axup, *Chaos in War: Is It Present and What Does It Mean?* Report to Air Command and Staff College, Maxwell AFB, Alabama, Academic Year 1994 Research Program, June 1994.

This is the best in-depth report examining *historical data* for evidence of Chaos. The authors find chaotic dynamics in tactical, operational, and strategic levels of military activity, examining data such as aircraft loss data for the entire Vietnam War, Allied casualty data in their advance through western Europe in World War II, and historical levels and trends in U.S. defense spending. The paper's greatest

strength is the discussion of data collection and analysis, the obstacles the authors encountered, and details of their search process. This full report is much more meaningful than the subsequent article they distilled for the *Airpower Journal* in late 1994.[123] Both the short article and the full essay contain some substantial technical errors in the basics of Chaos, but the authors have clearly done their homework.

T. Matsumoto et al., *Bifurcations: Sights, Sounds and Mathematics* (New York: Springer-Verlag, 1993).

This textbook generally expects the reader to have an extensive mathematical background, but it starts with a section describing simple electronic circuits, which exhibit a vast array of chaotic dynamics. This is a great reference for those with access to or interest in electronics applications. As its title implies, this book also includes a thorough study of various *classes of bifurcations* common to many dynamical systems.

Edward Ott, Tim Sauer, and James A. Yorke, eds., *Coping with Chaos: Analysis of Chaotic Data and the Exploitation of Chaotic Systems* (New York: John Wiley & Sons, 1994).

Topic-wise, this book is the best end-to-end compilation of chapters and articles, mostly published in other sources, which go from theoretical background to data analysis and applications. The text includes more recent work on practical suggestions for calculating dimensions, Lyapunov exponents, time embeddings, and control techniques. While the collection of articles is virtually all reprinted from primary sources, it is a good collection and can save an interested reader many hours of digging through periodical holdings. This book does require a solid background in vector calculus and differential equations, but it is very practical. The articles are generally at the level of papers from *Physical Review* and *Physical Review Letters*. The bibliography is extraordinary.

G. Mayer-Kress, ed., *Dimensions and Entropies in Chaotic Systems: Quantification of Complex Behavior*, Proceedings of an International Workshop at the Pecos River Ranch, New Mexico, 11–16 September 1985 (New York: Springer-Verlag, 1986).

This thin text offers the collection of papers contributed to the workshop cited. It is an older reference describing some of the early results of Chaos calculations. However, it presents a comprehensive review of techniques, modifications and improvements, and explanations of how they are related. The papers cover the intense details of how to calculate, in both theory and experiment, fractal measures, fractal dimensions, entropies, and Lyapunov exponents. This is a highly

technical work, not for the casual reader or weak of heart, and not a good place to first learn about these measurements. However, it *is* necessary reading for serious analysts embarking on numerical explorations of dynamical systems.

Michael F. Barnsley and Lyman P. Hurd, *Fractal Image Compression* (Wellesley, Mass.: AK Peters, 1993).

Perhaps more dense (i.e., slower) reading than Barnsley's first text, *Fractals Everywhere*, this fine book focuses appropriately on only those details required to understand the fractal compression techniques patented by Iterated Systems, Inc. It is a very thorough presentation, pleasant reading, and the text includes sample C source code and many demonstrations of decompressed images.

Saul Krasner, ed., *The Ubiquity of Chaos* (Washington, D.C.: American Association for the Advancement of Science, 1990).

This is another nice review of Chaos applications in a wide variety of disciplines: dynamical systems, biological systems, turbulence, quantized systems, global affairs, economics and the arms race, and celestial systems. Great bibliographies follow each individual article; most chapters have not been published elsewhere, as is often the case in similar collections of contributions by many independent authors.

*Naval Research Reviews*, Office of Naval Research, vol. XLV, no. 3, 1993.

This special issue is devoted to ONR-sponsored research in engineering applications of Chaos. Nice overview articles cover the following topics: controlling Chaos, noisy Chaos, communicating with Chaos, nonlinear resonance in neurophysiological systems, and image compression.

# VI

## Further Questions to Research

I have assembled in this chapter a broad collection of research topics that deserve more careful study. For the benefit of students and prospective research advisors, I have done my best to form the questions and issues into packages small enough to address within a short research term during in-residence professional military education.

*Complexity: The Next Big Step.* This report discusses how simple models can display complex behavior. However, once we develop a good intuition for Chaos, other questions arise immediately. Here is a peek at one of the central issues, only slightly oversimplified. *Fact:* fluids tend to move chaotically. The very nature of their dynamics makes them extremely sensitive to small disturbances. Now, the mixture inside a chicken egg is a fluid; that mixture is surely subjected to bumps and jostles during the formation of the baby chick inside. *Question:* if the fluid is chaotic, and its motion and behavior is so unpredictable, how does the creature inside always come out a chicken?

The answers to questions like these are the subject of the (even more recent) science of Complexity. You may consider researching complexity and self-organization. When and why do complicated systems sometimes organize themselves to behave "simply"? Which results of this theory are relevant for military decision makers?

*Exponents.* Identify a few specific military systems, perhaps within the context of a war game or through historical data, and calculate some Lyapunov exponents to compare the systems' relative sensitivity to perturbation. Prioritize the importance of various systems for protection or attack.

*Additional Dynamics.* Robert Axelrod's aggregation model successfully predicts the end states of two multi-party alliances, but there is still room to consider the *dynamics* of these alliances. How long do the alignments take to form? How stable are the end states? What sort of perturbations break the alliances? The analysis is static only, so far, although it does discuss the presence of "basins of attraction" of the end-state configurations.[124]

*Feedback.* Where are the feedback loops in current and future military systems? Consider both friendly and hostile systems. Also investigate both mechanical and social systems. Examine the strategic options for imposing feedback on these systems and protecting the systems from unwanted feedback. What behaviors and system transitions should we expect?

*Sensors.* What sort of sensors can we identify as vulnerable to imposed feedback? Where are they and how do they operate? What creative strategies can we devise to exploit or reduce their sensitivity to disturbances?

*War Games.* Can we replace random variables in war games with simple chaotic equations that produce comparable distributions? Can the underlying equations lead to clues about which parameters are most important? How do our games behave now? Can any be driven into Chaos with the right combination of parameters? For a detailed discussion of the use of historical data for battlefield predictions, see Colonel T.N. Dupuy's *Numbers, Predictions & War.*[125] It thoroughly discusses the issues of data compilation, modeling, prediction, and tabulates exhaustive lists of relevant battlefield parameters.

*The Nonlinear Battlefield.* Sean B. MacFarland, at the Army School of Advanced Military Studies (SAMS), defines "operational non-linearity" as the dispersed state of a combat force characterized by a complex of interconnecting fire positions and carefully sighted long-range weapons.[126] His paper highlights the difference between geometric nonlinearity and systemic (dynamical) nonlinearity. If we think of a force's physical disposition as its "state" in a combat system, old ideas of "forward edge of the battle area" may be replaced by emerging perspectives of overlapping attractors.

J. Marc LeGare, also at SAMS, proposed operations on the nonlinear battlefield organized in a "tactical cycle": disperse, mass, fight, redisperse, and reconstitute.[127] Could we structure this cycle to protect our own dynamics and take advantage of enemy cycles to break down their systems? If our forces are limited, can we exploit these cycles to apply our force efficiently and control the combat

dynamics? What kind of small perturbations could we impose on such a combat system? The answers to some of these questions may spring from other articles that consider the tactics of potential adversaries on the nonlinear battlefield.[128]

We should also note that the idea of dispersed, nonsequential operations is not new. In 1967, J.C. Wylie contrasted two very different kinds of strategies. One is *sequential*, a series of visible discrete steps that follow one another deliberately through time. The other is *cumulative*, "the less perceptible minute accumulation of little items piling one on top of the other until at some unknown point the mass of accumulated actions may be large enough to be critical." He observes that in the Pacific from 1941 to 1945 "we were not about to predict the compounding effect of the cumulative strategy (individual submarine attacks on Japanese tonnage) as it operated concurrently with and was enhanced by the sequential strategy [of the drive up the Pacific islands]."[129] Strategies like these may lend themselves to deeper analysis through Chaos theory.

## *Case Studies*

*For want of a nail the shoe is lost,*
*For want of a shoe the horse is lost,*
*For want of a horse the rider is lost,*
*For want of a rider the battle is lost,*
*For want of the battle the war is lost,*
*For want of the war the nation is lost,*
*All for the want of a horseshoe nail.*

George Herbert (1593–1632)

We already noted one effort to examine the *Mayaguez* crisis in the light of Chaos results. This was, of course, only a rough beginning. Several historical case studies (all entitled *For Want of a Nail!*) highlight the sensitivity of combat events to small "disturbances." The following references provide a list of candidate cases to consider for further Chaos analyses.

Robert Sobel composed a detailed counterfactual book of what would have happened had British General John Burgoyne held Saratoga in the American Revolution.[130] Hugh R. Wilson studied the ineffective application of economic sanctions against Italy in the winter of 1935–36 during the Italian military excursion into Ethiopia.[131] Hawthorne Daniel investigated the influence of logistics on war in several interesting case studies.

- American Revolution: New Jersey 1776; Lake Champlain and the Hudson River 1777
- Peninsular War: Spain and Portugal, 1808 to 1814
- The Moscow campaign: Russia, 1812
- American Civil War: 1861 to 1865
- Sudan Campaign: The upper Nile, 1896 to 1898
- The Allied invasion of occupied Europe, World War II: 1944 and 1945.[132]

*Bibliography.* With the recent explosion in Chaos resources, the preparation of a comprehensive bibliography would provide a great service to the general research community. The reference lists in the texts noted above are a good place to start. Many book reviews are also available to guide examinations of the most recent texts.

*Write More!* Above all, this essay should be regarded as one voice in a continuing conversation. Value always will be found through documenting other interesting thoughts and research. Please continue the conversation. In particular, there is plenty of room for open debate on issues this report has missed or overstated. It would also be most helpful for other reports to be published on additional military applications of which readers may be aware. I look forward to reading those reports.

# VII

## Conclusion

This report has focused on those issues of Chaos theory essential to military decision makers. The new science of Chaos studies behavior that is characterized by erratic fluctuations, sensitivity to disturbances, and long-term unpredictability. This paper has reviewed Chaos applications in military affairs and, I hope, corrected some deficiencies in current publications on Chaos.

The study was centered in three areas. First, we reviewed the fundamentals of chaotic dynamics to build some intuition for Chaos. Second, we surveyed the current military technologies that are prone to chaotic dynamics. Third, we saw how the universal properties of chaotic systems point to practical suggestions for applying Chaos results to strategic thinking and decision making. The power of Chaos comes from this universality: not just the vast number of chaotic systems but the common types of behaviors and transitions that appear in completely unrelated systems. As a result, recent recognition of Chaos in social systems offers new opportunities to apply these results to problems in decision making, strategic planning, and policy formulation.

The evidence is clear: chaotic dynamics pervade the dynamics of military affairs. The implications of Chaos theory offer an extraordinary range of options unavailable only twenty years ago. Not only do current military systems naturally exhibit chaotic dynamics, but many systems are vulnerable to new strategies that exploit Chaos results. Because of the theory's important potential, every military leader needs to be familiar with the fundamentals of Chaos in order to expect chaotic dynamics in military systems, recognize Chaos when it occurs, and exploit the vast array of tools for diagnosing and controlling those dynamics.

# Appendix

## What does it mean to be *Random*?

Our usual connotations of randomness carry images of erratic, completely unpredictable behavior. For a fair die on a craps table, *randomness* means that *sooner or later* that die will roll to a 6. It means there is no chance of that die rolling a string of 1's forever. If that were the case, the die would be very predictable, and thus, not random.

To be more precise, let us borrow an explanation from Batterman's article, "Defining Chaos."[133] Start with an infinite string of perfectly alternating digits:

01010101 ...

How much information does it take to recognize, transmit, or repeat this string? Suppose we had access to only a brief list of the first few elements of the sequence. Could we draw any conclusions about the system's behavior?

| | |
|---|---|
| 0 | No. |
| 0 1 | Not yet. |
| 0 1 0 | Hmmm, we begin to see a pattern. |
| 0 1 0 1 | Looks a little regular, but can't tell yet. |
| 0 1 0 1 0 | We can start to guess some regularity. . . . |

After 20, or 50, or 1,000 new pieces of information (additional digits in the observed string) we think we have it: this string of data has period two; we need only three pieces of information to repeat the string:

1. Print 0.
2. Print 1.
3. Repeat steps 1 and 2.

If we follow these steps, we're confident we can completely replicate the series. Now, if we do not know where or how the series was generated, we cannot be positive of its perfect periodicity. Nonetheless, as we get more and more information, our confidence in our analysis improves.

So how would we characterize a *random* string of data? In terms of our data string, it means we would need the ENTIRE infinite string—that is, an *infinite list*

*of instructions*—in order to accurately reproduce the original infinite data set. This requirement for an unending set of instructions to communicate or reproduce the data is sometimes offered as a formal definition of *randomness*.

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# About the Author

Glenn James entered the Service in 1982 as the Air Force Academy's top academic graduate. His four years as a cadet included a one-semester exchange at Ecole de l'Air, the French Air Force Academy. Proceeding directly to graduate school at the Georgia Institute of Technology, he earned a master's degree in mathematics, after which he took his first research assignment at the air Force Weapons Laboratory in Albuquerque, New Mexico.

In 1987, the Air Force Academy Department of Mathematical Sciences sponsored his return to Georgia Tech for a Ph.D., with a follow-on assignment as a mathematics instructor. In the Department of Mathematical Sciences, Captain James earned several teaching honors: the Tony M. Johnson Award for Excellence in Teaching Mathematics, the department's Outstanding Military Educator, and the academic rank of associate professor.

An early promotion to major led to his selection to attend the College of Naval Command and Staff at the Naval War College in Newport, Rhode Island. After graduating "with distinction," he was assigned to be Deputy Director of the Propulsion Sciences Division, the basic research arm of the Phillips Laboratory's Propulsion Directorate. He lives there now, at Edwards Air Force Base, California, with his wife Patricia and his children, Christine and Phillip.

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